

Staggered Wages and Prices in Dynamic Stochastic General Equilibrium Models

An Econometric Evaluation of the
Hybrid Wage New Keynesian Phillips Curve

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AN ECONOMETRIC EVALUATION OF THE
HYBRID WAGE NEW KEYNESIAN PHILLIPS CURVE

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To mom and dad
...and to my younger brother

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Staggered Wages and Prices in Dynamic Stochastic General Equilibrium Models,
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Abstract

This master's thesis at the Department of Economics, University of Oslo develops and estimates a Hybrid Wage New Keynesian Phillips Curve (Wage-NKPC) with staggered wages and prices using Norwegian data (provided by Statistics Norway). The different econometric methods discussed in the thesis are generalized method of moments (GMM), generalized instrumental variable estimation (GIVE) and Maximum Likelihood (ML) – in particular the Kalman-filter. The model is estimated using key macroeconomic variables such as gross domestic output, employment, wages, productivity, import prices and consumption.

All Stata-codes and Eviews-codes used in this thesis are available from the author upon request.

Keywords: New Keynesian Phillips Curve, generalized method of moments, generalized instrumental variable estimation, Kalman-filter, dynamic stochastic general equilibrium models

Preface

The experience of working with this thesis has been both stimulating and challenging. It has inspired me to pursue an academic career. A number of people deserve a special acknowledgement for their support and intellectual input since I started working on this thesis in January 2015. First and foremost, I want to extend my gratitude towards Professor Ragnar Nymoen, who has been my supervisor. Without his econometric and statistical expertise, attention to detail and several important advices and guidelines, this thesis would never have had the same end result. His guidance, suggestions and thorough supervision has been utmost invaluable. I would like to take this opportunity to thank Professor Ragnar Nymoen not only for his academic contribution to my master's thesis, but also for being a friend.

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This thesis would have never been the same without the insightful comments, corrections and contributions by the above-mentioned. To all of you, I am indebted.

I am solely responsible for any inaccuracies or errors still remaining in this thesis. None of the above-mentioned can be held responsible for any blunders I have made.

Oslo, May 2015

Herman Kruse

Contents

1	Introduction	1
2	Theoretical framework	3
2.1	Description of the model	3
2.2	Nominal wage rigidity	5
2.3	The Wage New Keynesian Phillips Curve	5
2.3.1	Operationalization of the desired wage rate	7
2.3.2	Joint dynamics of π_t^w and the desired wage rate w_t^*	9
2.3.3	A rational expectations solution for wage-inflation	10
2.4	Shortcomings	13
3	Econometric estimaton methods	14
3.1	The generalized instrumental variables estimator	14
3.2	Generalized method of moments	15
3.3	The Kalman-filter	17
3.3.1	ARMA(p,q) processes and maximum likelihood estimation	19
4	Empirical results	21
4.1	Data description	21
4.2	Application of GIVE to the Hybrid Wage-NKPC	22
4.3	Application of GMM to the Hybrid Wage-NKPC	25
4.4	Results of the GIVE approach	28
4.5	Results of the GMM approach	29
4.5.1	Actual versus fundamental inflation	30
4.6	Discussion of the forcing variables used in GIVE and GMM	33
4.6.1	Robustness checks	34
4.6.2	Unit root test of stationarity	39
4.7	Restricting the sum of coefficients on lead and lagged inflation	39
4.8	Application of the Kalman-filter to "desired wage rate"	42
4.8.1	Calibration of the Kalman-filter	42
4.9	Results of the Kalman-filter approach	43
4.9.1	Actual versus fundamental inflation with the Kalman-filter	44
4.10	Comparison of sectors	46
4.10.1	The total economy	46
4.10.2	The manufacturing sector	46

4.10.3 The public sector	47
5 Concluding remarks	47

List of Figures

1 Plot of all the variables used in the analysis, constructed as described above.	23
2 Time series plot of imported inflation and actual inflation.	24
3 Time series plot of the first-difference of the log of productivity in the manufacturing sector and the actual inflation.	25
4 Inflation: Actual vs. Fundamental, GMM method predicting fundamental inflation.	32
5 Inflation: Actual vs. Fundamental, GMM method predicting fundamental inflation. Restricted coefficients.	40
6 Inflation: Actual vs. Fundamental, Kalman-filter predicting fundamental inflation.	45

List of Tables

1 GIVE and GMM on the Wage-NKPC using annual data. Man-year denominator, full sample	27
2 Arellano-Bond test of autocorrelation	28
3 AR(1)-processes of the forcing variables	31
4 GIVE and GMM on the Wage-NKPC using annual data, adding additional lags as explanatory variables. Denominator man-year, full sample	36
5 Arellano-Bond test of autocorrelation, with additional lags of inflation as explanatory variables	37
6 GMM on the Wage-NKPC using annual data, robustness checks. Denominator man-year, subsamples sample	38
7 Unit root test of stationarity	39
8 GMM and GIVE on the Wage-NKPC using annual data, restricted coefficients. Denominator man-year, full sample	41
9 Kalman-filter on the AR(1) process of the desired wage	43

10	Kalman-filter on the Wage-NKPC using annual data. Denominator man-year, full sample	44
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Appendices

Appendix A - Output for the Mainland Economy	52
Appendix B - Output for other sectors	58

1 Introduction

The study of macroeconomic fluctuations has been a key interest for several economists over the last century. Dynamic stochastic general equilibrium (DSGE) models are by many considered the state-of-the-art models. Because the models are built on microeconomic foundations, they are able to explain important decision making processes down to individual levels. The models are thus able to generate quantitative predictions about macroeconomic fluctuations and about the behavior of the underlying shocks. The aggregation over individual behavior typically leads to a link between short run inflation and overall real activity in the economy.¹

Starting with the seminal paper of Kydland and Prescott (1982), which introduced the concept of multi-period production cycles and introduced a general equilibrium to the existing growth and business cycle theory, the DSGE literature has since become the state-of-the-art models of macroeconomic fluctuations. Several papers have contributed towards the formulation of a benchmark model. The ultimate goal is to build a model of fluctuations which includes all the strengths of the so-far proposed models combined. However, there is no consensus about the ingredients that are critical to include in such a model. For instance Erceg, Henderson and Levin (2000), Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005) all contributed to the standing literature with different variations of the DSGE models. The result has often been quite complicated models, but with absence of pure empirical evidence of relevance for the macroeconomic fluctuations.

Most of the literature focuses on staggered price setting. This thesis will instead consider staggered wages, and will use a number of econometric methods to evaluate the hybrid Wage New Keynesian Phillips Curve (Wage-NKPC) using annual Norwegian data. Emerging from the recent interest of several economists, such as Galí and Gertler (1999), in the dynamics of short-run inflation, early work by for instance Fischer (1977), Taylor (1980) and Calvo (1983) have given important advances in the theoretical modeling of staggered nominal price and wage setting. In line with the microfoundation, the proposed theoretical framework by these economists and others cast the staggered prices and wages into an individual optimization problem.

¹Of course, it is possible to debate the microfoundation. For instance, one could question the microfoundation because the models rely on a simplifying assumption of self-interested hyper-rational agents and no coordination problem.

This thesis offers two distinctive features. First, it develops and estimates a hybrid Wage New Keynesian Phillips Curve with staggered nominal wage and price setting using a Norwegian data set, embedding Calvo-style staggered prices and wages. Second, it uses three econometric modeling techniques to provide detailed insight into the dynamics of the model. I use generalized instrumental variables estimation (GIVE), generalized method of moments (GMM) and the Kalman-filter, all econometric tools which will be discussed in detail in Section 3. The first two are the main tools used in most of the existing literature, see for instance Galí and Gertler (1999). The third tool, namely the Kalman-filter, is a tool which just recently has had an upswing in popularity due to improved computer software. It involves an iterative process using maximum likelihood techniques, and proves extremely valuable in the modeling of unobserved components which so often appear in macroeconomic models. Hence, the importance of this contribution to the literature of macroeconomic fluctuations can be paramount.

The findings in this thesis are several. I find that the important coefficients of lead and lagged inflation are statistically significant and robust with a positive sign, forward-dominance and are not jointly rejected as summing to one, all according to theory. These results apply to all the estimation methods, which give strong support to the model. Furthermore the GMM procedure generally performs better than GIVE on the model because of considerable autocorrelation and the more general structure of the GMM residual correlation matrix. Noteworthy is that some of the coefficients are estimated with opposite sign of the theoretical ones and are insignificant at conventional levels. However, none of the specifications or the estimation methods jointly reject the forcing variables. I find that the dynamic system consisting of inflation forcing variables has a stable stationary solution in both GIVE and GMM with no unit roots and no moduli greater than one. On the other hand, by calculating the rational expectations solution based on the standard assumption about exogenous forcing variables, I am able to plot fundamental and actual inflation. Robustness checks show that the model is robust to several alternative specifications. The Kalman-filter returns a significant coefficient on the unobservable component in the model, the "desired wage rate", which is a quite strong result. The Kalman filter also returns coefficients on the lead and lagged inflation which sums roughly to one and involves forward dominance. The Kalman-filter shows a reasonable pattern when plotting fundamental and actual

inflation. The thesis shows that this recursive maximum likelihood principle succeeds in modeling an unobservable component with reasonable parameter values. Thus, this thesis will show that the wage version of the price-NKPC can also give accurate predictions about the macroeconomic dynamics of inflation. It also suggests the Kalman-filter as an improved method to most of the literature, which mainly focus on GMM.

The remainder of this thesis is organized as follows. In Section 2, I describe the model framework and derive some of the relationships that will be key to the modeling of the staggered wages. In particular, I describe how the nominal wage rigidity will enter and derive the hybrid Wage New Keynesian Phillips Curve. In Section 3, I establish some econometric methodology necessary for the modeling and empirical work. In Section 4, I report the results of the empirical studies and analyze them. Section 5 concludes.

2 Theoretical framework

This section will describe the model, the economy and the framework that will be used in this thesis. First, I establish the description of the economy and describe the behavior of the agents. Then I describe how this model differs from the standard literature, in particular by describing how the nominal wage rigidity enters.² Finally, I discuss the hybrid Wage New Keynesian Phillips Curve and how this thesis will cope with its unobservable components.

2.1 Description of the model

The economy is assumed to consist of utility maximizing households who offer (monopolistically) their specialized labor supply and consume the final goods produced by firms. The production side consists of monopolistically competitive firms employing the specialized labor service to produce an intermediate good used in the production of a final good sold in a competitive market. Furthermore, there is a policy making authority who conducts monetary policy according to some rule set. In the literature, it is established that nominal rigidities such as barriers to price adjustment can cause monetary changes to have real effects (Romer 2012, ch.

²See for instance Galí and Gertler (2007), Goodfriend (2007) or Mankiw (2006) for a historical overview of the consensus on the "new neo-classical synthesis". See Clarida, Galí and Gertler (1999) for a more analytical evaluation of the synthesis.

7). In the macroeconomic model that I will refer to in my econometric modeling of wages, the main nominal rigidities will be staggered price and wage setting, leading to dynamic (rather than instantaneous) adjustment of price and wage levels. There are two reasons for this assumption. First, standard economic theory indicates a relationship between wage changes and inflation. Hence, modeling the inflation fluctuations with wage changes as the forcing variable should be equally important to the modeling using prices as the forcing variable. Second, since most changes to wages in Norway take place on an annual basis, it is clearly worth considering wages to be staggered across time. An immediate implication of this is that wages are not only state dependent, but also time dependent. The latter shall be a main focus in the empirical testing of the relevance of staggered wages using general method of moments and generalized instrumental variable estimation. The former shall be important when using the Kalman-filter method, which will require the model to be specified on the state space form.

In the model, the economy is assumed to consist of a fixed number of infinitely lived households obtaining utility from consumption and disutility from labor. They follow standard textbook utility maximization for which the complete derivation is readily available elsewhere, see for instance Romer (2012). Firms are owned by the households, and produce according to a production function with labor as the only input. They follow standard textbook optimization for which the complete derivation is readily available elsewhere, see for instance Romer (2012).

I assume that prices and wages are contracted over a period of time, using so-called Calvo-pricing, cf. Calvo (1983). This means that the privilege of adjusting nominal wage and prices from one period to the next is a random event with a constant probability. It is implied that the marginal cost will be stochastic since it is dependent of the current wage rate.

It remains to explain the behavior of the policy making authority (the central bank), which determines the real interest rate. In practice, the central bank sets the nominal interest rate, but if we assume that the inflation expectations follow the forecast of the central bank, the central bank can at least set the expected real interest rate. In the model, I will assume that this is the case. A standard assumption is that the central bank follows some rule for how it sets the real interest rate as a function of macroeconomic conditions. For instance, the central

bank can have a target path for the GDP and conducts monetary policy to achieve that target. The determination of the real interest rate will not be a main aspect in this thesis, however, and for further reading see for instance Clarida, Galí and Gertler (1999) or Romer (2012).

2.2 Nominal wage rigidity

There are a number of different ways to implement wage rigidity. One classical way of implementing staggered wages is the Taylor model, cf. Taylor (1979). Taylor used a model framework where wage contracts are set for two periods at a time, and where it is known that the contract will be renegotiated when those two periods have passed. This deterministic approach can be both fruitful and highly tractable; however it is perhaps not very realistic. Another way of implementing staggered wages is to use the Calvo model, Calvo (1983). Calvo assumes that instead of a fixed deterministic number of periods between wage settings, the privilege of renegotiating wages will be given stochastically. More specifically, the opportunity will follow a Poisson process, and the probability to renegotiate will therefore be assumed to be constant across time periods. This means that the probability of being allowed to renegotiate the wage is the same regardless of how many periods have passed since the last negotiation. The importance of the Calvo assumption is twofold. First, the degree of price stickiness can easily be altered by changing the parameter value, i.e. the probability. Second, it leads to a tractable derivation of the hybrid Wage New Keynesian Phillips Curve with staggered wages.

Wages are set using the Calvo mechanism, so that every period $\theta \in (0, 1)$ households randomly drawn from the population, are allowed to re-optimize their wage rate.

2.3 The Wage New Keynesian Phillips Curve

In this section and the rest of the thesis, bold-fonted variables are matrices and the operator \mathbb{E} denotes the expectations-operator. A superscript $^\top$ denotes the transposed of a matrix.

In most of the established literature, one uses staggered prices instead of staggered wages. In each period, a fraction $\theta \in (0, 1)$ of firms can set new prices with those firms chosen at random, see for instance Romer (2012, ch. 7.4).

Instead of focusing on staggered prices, the focus in this model shall be staggered wages. A main difference from the literature focusing on staggered prices is that households now hold some market power in setting wages for the differentiated labor services they supply. The households supply monopolistically their specialized type of labor. Firms now only decide how many working hours they want given the wage rate set by households. The relationship which will be studied allows for some degree of backward-looking wage setting, which nests the original NKPC as a special case (in particular when $\alpha_b = 0$ in (5)). Following Galí and Gertler (1999), the wage-NKPC can be derived using the following relationships:

Let the logarithm of the aggregate wage (w_t) evolve according to:

$$w_t = \theta w_{t-1} + (1 - \theta) \bar{w}_t^* \quad (1)$$

Where \bar{w}_t^* is the index for the wage newly set in period t . Then let w_t^f be the wage set by a forward-looking household, and w_t^b be the wage set by a backward-looking household. We can then write the index as:

$$\bar{w}_t^* = (1 - \omega) w_t^f + \omega w_t^b \quad (2)$$

According to the Calvo-model, the forward-looking households set the wage according to:

$$w_t^f = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_t \{ w_{t+k}^* - w_{t+k} \} \quad (3)$$

The structural term β is the households' subjective discount factor. This means that the forward-looking households set their wage rate knowing the probability of being randomly selected to reoptimize in the future. The backward-looking households set the wage according to:

$$w_t^b = \bar{w}_{t-1}^* + \pi_{t-1}^w \quad (4)$$

Which says that the backward-looking households set the wage equal to the most recent optimal wage adjustment with a correction for the wage inflation.

Combining these rules lead to the following hybrid wage New Keynesian Phillips Curve (hereafter Wage-NKPC) relationship, see Brubakk and Sveen (1/2009), in

logarithmic terms:

$$\pi_t^w = \alpha_b \pi_{t-1}^w + \alpha_f \mathbb{E} \pi_{t+1}^w + \tau(w_t^* - w_t) + \varepsilon_{\pi t}^w \quad (5)$$

Where $\alpha_b, \alpha_f, \tau \geq 0$. The wage growth (inflation) is dependent on lagged inflation and expected future inflation, i.e. the inflation rate in period $t+1$ forecasted using information available at the end of period t . Wage growth is also driven by the difference in "desired wage rate" by the households, which hold some market power in setting the wages, and the actual wage rate at the current time. The labor market is characterized by monopolistic competition. By the "desired wage rate", I mean the wage level that would be realized if wages were fully flexible. In a precise and theoretical sense, it is the marginal rate of substitution between consumption and leisure, so that it measures the households' loss of utility in terms of consumption units from providing one extra unit of labor supply.

Note that some of the existing literature operates with the "exact form" of this equation, where the error term $\varepsilon_{\pi t}^w$ is omitted. For the purpose of this thesis, however, we stick to this form where the interpretation of the error term has relevance in the econometric modeling. The "hybrid" term is characterized by $\alpha_b > 0$. In the rest of the paper, it will be assumed that the model is on the hybrid form because assuming $\alpha_b = 0$ implies unrealistically low inflation persistence.

The hybrid New Keynesian Phillips curve includes a backward-looking term of inflation, so that $\alpha_b > 0$.

2.3.1 Operationalization of the desired wage rate

The "desired wage rate" is not directly observable, since it is a highly theoretical measure. Hence, one important aspect of the thesis will be how we should operationalize the "desired wage rate" w_t^* using econometric methodology. Since the forcing variable in the presented model is the difference in desired and current wage rate, how we estimate the desired wage rate is key. Wage growth is driven by the difference in the desired wage rate and the actual wage compensation in the current time period. The options of how to operationalize this theoretical term are several, and this thesis most likely does not cover all of them. I will, however, come up with some suggestions and pursue them empirically.

To cope with the problem of operationalizing the unobservable desired wage rate, we can restate the model in a more ad hoc manner which includes a "pressure indicator" and "ability to pay wages", cf. Brubakk and Sveen (1/2009). In this approach, I use a linear combination of private consumption, change in private consumption between periods and employment level as the pressure indicator. We can also note that the actual wage paid is the sum of the real marginal costs and the productivity. In this sense, the change in wages from one period to another is a combination of the households' pressure indicator and the firms' ability to pay wages. This gives us the following equation for the wage growth:

$$\pi_t^w = \alpha_b \pi_{t-1}^w + \alpha_f \mathbb{E}_t \pi_{t+1}^w + \tau [\phi c_t + \psi \Delta c_t + \gamma l_t - \kappa(y_t - l_t) - \zeta mc_t] + \varepsilon_{\pi t}^w \quad (6)$$

Where the first three terms in the brackets constitute the "pressure indicator" or the households' negotiation power, and the last two are the ability to pay wages by the firms, and $\alpha_b, \alpha_f, \tau, \phi, \psi, \gamma, \zeta \geq 0, \kappa \leq 0$. Now the forcing variables are the consumption level, the change in consumption level, productivity and the marginal cost. The theory suggests a strong contemporaneous correlation between marginal cost and inflation. This suggests that the marginal cost should enter the structural equation significantly. It is of course possible to question this re-parameterization of the model. Even if we reparameterize in this way, some of the components in (6) are not directly observable either. For instance, the marginal cost mc_t must be proxied using some other measure. Even if we could observe all the components directly, it is still possible to question the link between these components and the theoretical term "desired wage rate". Thus, finding parameter values for (6) does not necessarily reflect a solution to (5). Even if we can question the reparameterization (6) and its relevance for the modeling of (5), it is still worthwhile to pursue (6) and find parameter values. This will be done in Section 4.

Another way of operationalizing the desired wage rate, besides the reparameterization proposal (6), is to use a *Kalman-filter*. This will allow us to use the model (5), where the unobservable marginal rate of substitution between consumption and leisure can be "extracted" from observables using maximum likelihood principles. This is a iterative process where information about the process will recursively added and updated. Provided that the noise in the observed variables is Gaussian white noise, a Kalman-filter is optimal in the sense that it minimizes the mean square error of the estimated parameters. If the noise is not Gaussian white noise,

then the Kalman-filter is the best linear estimator, but there may be non-linear estimators which are better. We shall assume that the noise is Gaussian white noise and proceed using a Kalman-filter to estimate our unobserved component in the "desired wage rate".

2.3.2 Joint dynamics of π_t^w and the desired wage rate w_t^*

Galí and Gertler (1999) estimate a hybrid price-NKPC using generalized method of moments (GMM). They find that models where inflation is a function of expected future inflation and real marginal costs today is a good approximation of inflation dynamics in the U.S. and Europe. According to Nymoen et al. (2012), the typical empirical result for the hybrid-NKPC is that forward-dominance is supported by data, i.e. $\alpha_f \geq \alpha_b$. Typically, the hypothesis of $\alpha_f + \alpha_b = 1$ is not rejected at conventional levels of significance, which is also theoretically implied if $\beta = 1$, cf. Galí and Gertler (1999). The literature establish a number of ways to model the term $\mathbb{E}_t \pi_{t+1}^w$. One simple possibility is to propose rational expectations in the sense that:

$$\mathbb{E}_t \pi_{t+1}^w = \pi_{t+1}^w + \eta_{t+1} \quad (7)$$

This will allow us to rewrite the Wage-NKPC in (5) as:

$$\pi_{t+1}^w = \frac{1}{\alpha_f} \pi_t^w - \frac{\alpha_b}{\alpha_f} \pi_{t-1}^w - \frac{\tau}{\alpha_f} (w_t^* - w_t) - \frac{1}{\alpha_f} \varepsilon_{\pi t}^w - \eta_{t+1} \quad (8)$$

This method will allow us to estimate the model using non-linear least squares (NLS), at least provided $\alpha_f \neq 0$. The following relationship suggests that the desired wage rate is a backward-looking relationship where there may be feedback from inflation. The relationship is proposed for the purpose of illustrating the joint dynamics of the wage inflation and the desired wage rate.

$$w_t^* - w_t = v \pi_{t-1}^w + \rho (w_{t-1}^* - w_{t-1}) + \varepsilon_{wt} \quad (9)$$

Following Bårdsen et al. (2004)³ we get that these two equations (8) and (9) have the characteristic polynomial:

$$\pi^w(\lambda) = \lambda^3 - \left[\frac{1}{\alpha_f} + \rho \right] \lambda^2 + \frac{1}{\alpha_f} [\alpha_b + \tau v + \rho] \lambda - \frac{\alpha_b}{\alpha_f} \rho \quad (10)$$

³They use this setup in a framework with price inflation and with marginal cost as the forcing variable.

There exists a stationary solution if and only if none of the three roots are on the unit circle. If we follow Bårdsen et al. (2004) and use the theoretically suggested coefficients $\alpha_b = 0.25, \alpha_f = 0.75, v = 0, \rho = 0.7$ we get the roots $\{3.0, 1.0, 0.7\}$, which suggests that there is no (stable) stationary solution for the two variable system consisting of wage inflation and the gap between the desired wage rate and the actual wage. This can be imposed on the system by restricting $\alpha_f + \alpha_b = 1$. With this homogeneity assumption, the forcing variable thus has to have an equilibrating mechanism for the system to be stationary, and the v cannot be zero in this particular model.

Another way of operationalizing the forward-term is to use instrumental variables to compute a proxy for the forward-term $\mathbb{E}_t \pi_{t+1}^w$. Since we know that adding instruments will numerically move instrumental variable (IV) estimation towards ordinary least squares (OLS), which is by the *Gauss-Markov theorem* the best linear unbiased estimator (BLUE), using the IV-method can be both fruitful and lead to the best result. Using this method also allow us to disregard the way the households form their expectations, and rather use proxies to model their expectations. This will be discussed in detail in section 3 and applied to the model in Section 4.

2.3.3 A rational expectations solution for wage-inflation

The following section uses repeated substitution to find the rational expectations solution of the Wage-NKPC. The data generating process leading to the brute force solution of (6) needs to define the process of all the forcing variables. Note that this solution assumes strongly exogenous forcing variables. This assumption may be unrealistic, but necessary to display the rational expectations solution. However, this assumption follows the seminal work of Galí and Gertler (1999) who applied a rational expectations solution to the price-NKPC and emphasized a high degree of fit for the NKPC in the US. The forcing variables are strongly exogenous if they are at least weakly exogenous in (6) and π is not Granger-causing the forcing variables, which means π_{t-1} does not affect any of the forcing variables. This may not be consistent with the idea that inflation is a variable that feed-back to several important macroeconomic variables. The entire system, including (6),

is presented for completeness:

$$\pi_t^w = \alpha_b \pi_{t-1}^w + \alpha_f \mathbb{E}_t \pi_{t+1}^w + \tau [\phi c_t + \psi \Delta c_t + \gamma l_t - \kappa(y_t - l_t) - \zeta m c_t] + \varepsilon_{\pi t}^w \quad (11)$$

$$c_t = \rho_c c_{t-1} + \varepsilon_{ct} \quad (12)$$

$$\Delta c_t = (\rho_c - 1) c_{t-1} + \varepsilon_{ct} = \rho_{\Delta c} c_{t-1} + \varepsilon_{ct} \quad (13)$$

$$l_t = \rho_l l_{t-1} + \varepsilon_{lt} \quad (14)$$

$$A_t = \rho_A A_{t-1} + \varepsilon_{At} \quad (15)$$

$$m c_t = \rho_{mc} m c_{t-1} + \varepsilon_{mct} \quad (16)$$

Where we let $A_t = y_t - l_t$ and, without loss of generality, let $\tau = 1$ since it is a multiplicative positive constant which may very well be incorporated into the other coefficients. Following Bårdsen et. al (2005, Appendix A.2.1) with some difference in notation, we start by getting rid of the lagged dependent variable by implicitly defining a new variable $\tilde{\pi}_t = \pi_t - \tilde{\alpha}_b \pi_{t-1}$. Then apply the expectation one period ahead:

$$\begin{aligned} \mathbb{E}_t \pi_{t+1}^w &= \mathbb{E}_t \tilde{\pi}_{t+1}^w + \tilde{\alpha}_b \mathbb{E}_t \pi_t^w \\ \mathbb{E}_t \pi_{t+1}^w &= \mathbb{E}_t \tilde{\pi}_{t+1}^w + \tilde{\alpha}_b \tilde{\pi}_t^w + \tilde{\alpha}_b^2 \pi_{t-1}^w \end{aligned} \quad (17)$$

Then we substitute for (17) into (6) to get:

$$\begin{aligned} \tilde{\pi}_t^w + \tilde{\alpha}_b \pi_{t-1}^w &= \alpha_f (\mathbb{E}_t \tilde{\pi}_{t+1}^w + \tilde{\alpha}_b \tilde{\pi}_t^w + \tilde{\alpha}_b^2 \pi_{t-1}^w) + \alpha_b \pi_{t-1}^w \\ &+ [\phi c_t + \psi \Delta c_t + \gamma l_t - \kappa(y_t - l_t) - \zeta m c_t] + \varepsilon_{\pi t}^w \end{aligned} \quad (18)$$

$$\begin{aligned} \tilde{\pi}_t^w &= \left(\frac{\alpha_f}{1 - \alpha_f \tilde{\alpha}_b} \right) \mathbb{E}_t \tilde{\pi}_{t+1}^w + \left(\frac{\alpha_f \tilde{\alpha}_b^2 - \tilde{\alpha}_b + \alpha_b}{1 - \alpha_f \tilde{\alpha}_b} \right) \pi_{t-1}^w \\ &+ \left(\frac{1}{1 - \alpha_f \tilde{\alpha}_b} \right) [\phi c_t + \psi \Delta c_t + \gamma l_t - \kappa(y_t - l_t) - \zeta m c_t] + \left(\frac{1}{1 - \alpha_f \tilde{\alpha}_b} \right) \varepsilon_{\pi t}^w \end{aligned} \quad (19)$$

We have defined the parameter $\tilde{\alpha}_b$ as:

$$\tilde{\alpha}_b^2 - \frac{1}{\alpha_f} \tilde{\alpha}_b + \frac{\alpha_b}{\alpha_f} = 0 \quad (20)$$

Which has the solution:

$$\tilde{\alpha}_{b,i} = \frac{1 \pm \sqrt{1 - 4\alpha_f\alpha_b}}{2\alpha_f}, \quad i \in \{1, 2\} \quad (21)$$

Where the stable backward solution is characterized by $|\tilde{\alpha}_{b,i}| < 1$ for either $i = 1$ or $i = 2$. We then have a pure forward looking model:

$$\begin{aligned} \tilde{\pi}_t^w = & \left(\frac{\alpha_f}{1 - \alpha_f\tilde{\alpha}_b} \right) \mathbb{E}_t \tilde{\pi}_{t+1}^w + \left(\frac{1}{1 - \alpha_f\tilde{\alpha}_b} \right) \\ & \cdot [\phi c_t + \psi \Delta c_t + \gamma l_t - \kappa(y_t - l_t) - \zeta m c_t] + \left(\frac{1}{1 - \alpha_f\tilde{\alpha}_b} \right) \varepsilon_{\pi t}^w \end{aligned} \quad (22)$$

Then, by imposing a unit root (by forcing $\alpha_f + \alpha_b = 1$), we have that $\tilde{\alpha}_{b,1} + \tilde{\alpha}_{b,2} = \frac{1}{\alpha_f}$ so that the model becomes:

$$\begin{aligned} \tilde{\pi}_t^w = & \left(\frac{1}{\tilde{\alpha}_{b,2}} \right) \mathbb{E} \pi_{t+1}^w + \left(\frac{1}{\alpha_f \tilde{\alpha}_{b,2}} \right) \\ & \cdot [\phi c_t + \psi \Delta c_t + \gamma l_t - \kappa(y_t - l_t) - \zeta m c_t] + \left(\frac{1}{\alpha_f \tilde{\alpha}_{b,2}} \right) \varepsilon_{\pi t}^w \end{aligned} \quad (23)$$

Which can be written compactly as:

$$\tilde{\pi}_t^w = \beta_f \mathbb{E}_t \tilde{\pi}_{t+1}^w + \beta_c c_t + \beta_{\Delta c} \Delta c_t + \beta_l l_t - \beta_A (y_t - l_t) - \beta_{mc} m c_t + v_{\pi t}^w \quad (24)$$

Then the solution can be found by finding $\mathbb{E}_t \tilde{\pi}_{t+1}^w$ and then solve for $\tilde{\pi}_t^w$. The intermediary steps are cumbersome and uninformative. An alternative method is to use the method of undetermined coefficients, which will lead to the same result. Both methods are presented in Bårdsen et. al (2005, Appendix A.2.1 and A.2.2). The end result is the following:

$$\begin{aligned} \pi_t^w = & \tilde{\alpha}_{b,1} \pi_{t-1}^w + \left(\frac{\phi}{\alpha_f(\tilde{\alpha}_{b,2} - \rho_c)} \right) c_t + \left(\frac{\psi}{\alpha_f(\tilde{\alpha}_{b,2} - \rho_{\Delta c})} \right) \Delta c_t + \left(\frac{\gamma}{\alpha_f(\tilde{\alpha}_{b,2} - \rho_l)} \right) l_t \\ & - \left(\frac{\kappa}{\alpha_f(\tilde{\alpha}_{b,2} - \rho_A)} \right) A_t - \left(\frac{\zeta}{\alpha_f(\tilde{\alpha}_{b,2} - \rho_{mc})} \right) m c_t + \left(\frac{1}{\alpha_f \tilde{\alpha}_{b,2}} \right) \varepsilon_{\pi t}^w \end{aligned} \quad (25)$$

By using this solution to the system we can plot what is often referred to in the literature as fundamental inflation and compare to actual inflation, as in Galí and Gertler (1999). We can then evaluate whether fundamental inflation tracks the behavior of actual inflation well. Naturally, this solution can be applied with

the estimation results from both GIVE and GMM. The solution for the model in (5) which relies on maximum likelihood and the Kalman-filter follows the same pattern, but with a more compact solution equation:

$$\pi_t^w = \tilde{\alpha}_{b,1}\pi_{t-1}^w + \left(\frac{1}{\alpha_f(\tilde{\alpha}_{b,2} - \rho)}\right)(w_t^* - w_t) + \left(\frac{1}{\alpha_f\tilde{\alpha}_{b,2}}\right)\varepsilon_{\pi t}^w \quad (26)$$

2.4 Shortcomings

There are a number of shortcomings to these approaches which should be mentioned. First and foremost, due to the unobservability of the desired wage rate, the conventional measures will be ridden with error. Even if we use instrumental variables, Kalman-filter or the suggested re-parameterization into a pressure indicator and ability to pay wages, the estimation is still likely to involve a considerable measurement error. We have to take into account that we might not fully cover the way households form their desire of some wage rate. The link between households desired wage rate and the pressure indicator and ability to pay wages can be weak (or at least not strong). Another issue is that even if we could observe the desired wage rate exactly, the link between this term and the marginal cost of the firms could have limited support in the data. Movements in marginal costs do not necessarily have to be met by co-movements in real wages, and thus the ability to pay wages-term can in itself have limited support. If this is true, then this will likely result in poor estimation results and support in the data for the NKPC.

The model relies on a simplifying assumption to avoid interdependency between price and wage setting decisions, cf. Carlsson and Westermarck (2011). Introducing both Calvo-type staggered prices and wages when wages are set within the firm is complicated, because there will be a dependency between current and future wage and price decisions. If the firm changes prices today, it affects both current and future profits. This will also affect the future wage setting through the firm's future surpluses. This, then, will affect the firm's marginal cost, leading to changes in optimal prices. Hence, we get a dependency between price and wage setting. This will be a problem for all models where price and wage setting occurs within the same sector. However, following Gertler et al. (2008), we can separate price and wage setting into different sectors, so that households decide their wage rate for their specialized labor supply and firms decide their optimal price level.

3 Econometric estimation methods

The following proposed estimation methods are meant to cope with the two problems of an unobservable lead in inflation and the operationalization of the desired wage rate. Because the lead inflation is not directly observable, we use econometric methods to proxy for this variable when estimating the model. Without the re-parameterization proposal (6), the model (5) demands a direct modeling of the unobservable desired wage rate. This section proposes the Kalman-filter as a solution.

3.1 The generalized instrumental variables estimator

The relationship we want to estimate is a special case of the following linear model:

$$\mathbf{y} = \mathbf{X}\beta + \mathbf{u}, \quad \mathbb{E}(\mathbf{u}\mathbf{u}^\top) = \sigma^2\mathbf{I} \quad (27)$$

We need at least one entry in \mathbf{X} not to be predetermined with respect to the error terms. For various reasons, it is very often so that a list of l variables suggest themselves as relevant IV estimators for the predetermined regressors. Let \mathbf{W} denote an $n \times l$ matrix of instruments, and let there be $k < l$ number of regressors in the model. We then have overidentification because we in general can formulate the moment conditions in more than one way. This, however, is not a problem, since we can always choose exactly k linear combinations of the l columns of \mathbf{W} and treat the system as just identified. We then seek the optimal $l \times k$ matrix \mathbf{J} such that \mathbf{WJ} is a valid instrument matrix and such that the asymptotic covariance matrix obtained using \mathbf{WJ}^* is minimized using \mathbf{J} . This asymptotic covariance matrix of the IV estimator using \mathbf{WJ} as an instrument is, following Davidson and Mackinnon (2009, ch. 8.3):

$$\sigma_0^2 \text{plim}_{n \rightarrow \infty} (n^{-1} \bar{\mathbf{X}}^\top \mathbf{P}_{\mathbf{WJ}} \bar{\mathbf{X}})^{-1} \quad (28)$$

where $\mathbf{P}_{\mathbf{WJ}}$ is an orthogonal projection matrix. An orthogonal projection matrix is an idempotent ($\mathbf{P}_{\mathbf{WJ}} = \mathbf{P}_{\mathbf{WJ}}^2$) matrix which maps a vector to one specific point in the plane.

We can test the overidentifying restrictions by using a test statistic based on the IV criterion function. For any just identified model, the IV residuals are orthogonal to the full set of instruments. A test based on the criterion function is often

called a Sargan-Hansen test after Sargan (1958). The Sargan-Hansen test may reject the null hypothesis if the model is misspecified, one or more instruments are invalid, some instruments may be regressors or the finite sample distribution is substantially different from the asymptotic distribution.

A natural solution to the problem of finding the best solution of the instrument matrix is to project $\bar{\mathbf{X}}$ orthogonally on to the space $\mathcal{S}(\mathbf{W})$, which yields the instrument matrix $\mathbf{WJ} = \mathbf{P_W}\bar{\mathbf{X}} = \mathbf{W}(\mathbf{W}^\top \mathbf{W})^{-1} \mathbf{W}^\top \bar{\mathbf{X}}$, which immediately implies that:

$$\mathbf{J} = (\mathbf{W}^\top \mathbf{W})^{-1} \mathbf{W}^\top \bar{\mathbf{X}} \quad (29)$$

If we use $\mathbf{P_W}\mathbf{X}$ as the matrix of instrumental variables, the moment condition defining our GIV estimator is $\mathbf{X}^\top \mathbf{P_W}(\mathbf{y} - \mathbf{X}\beta) = 0$, which can be solved to yield our GIV estimator:

$$\hat{\beta}_{\text{GIVE}} = (\mathbf{X}^\top \mathbf{P_W}\mathbf{X})^{-1} \mathbf{X}^\top \mathbf{P_W}\mathbf{y} \quad (30)$$

3.2 Generalized method of moments

The relationship we want to estimate is a special case of the following linear model:

$$\mathbf{y} = \mathbf{X}\beta + \mathbf{u}, \quad \mathbb{E}(\mathbf{u}\mathbf{u}^\top) = \mathbf{\Omega} \quad (31)$$

There are n observations, and $\mathbf{\Omega}$ is the $n \times n$ covariance matrix. The main difference in (31) from (27) is that we in (31) assume a general covariance matrix where variances are allowed to be different and covariance between the residuals can be present. In (27), we instead assumed constant variance and no covariance between the residuals, in particular we assumed that the covariance matrix was proportional to the identity matrix. Some of the variables in the $n \times k$ matrix \mathbf{X} may not be predetermined with respect to the error terms \mathbf{u} . We assume that there exists a $n \times l$ matrix of predetermined instrumental variables \mathbf{W} with $n > l$ and $l \geq k$, such that $\mathbb{E}(u_t | \mathbf{W}_t) = 0$. We assume that for all $t, s = 1, \dots, n$ we have $\mathbb{E}(u_t, u_s | \mathbf{W}_t \mathbf{W}_s) = \omega_{ts}$, where ω_{ts} is the ts^{th} element of $\mathbf{\Omega}$. Following Davidson and Mackinnon (2009, ch. 9.2) we get that:

$$\text{var}(n^{-1/2} \mathbf{W}^\top \mathbf{u}) = \frac{1}{n} \mathbb{E}(\mathbf{W}^\top \mathbf{u} \mathbf{u}^\top \mathbf{W}) = \frac{1}{n} \sum_{t=1}^n \sum_{s=1}^b \mathbb{E}(u_t u_s \mathbf{W}_t^\top \mathbf{W}_s) \quad (32)$$

$$= \frac{1}{n} \sum_{t=1}^n \sum_{s=1}^b \mathbb{E}(\mathbb{E}(u_t u_s \mathbf{W}_t^\top \mathbf{W}_s | \mathbf{W}_t, \mathbf{W}_s)) \quad (33)$$

$$= \frac{1}{n} \sum_{t=1}^n \sum_{s=1}^b \mathbb{E}(\omega_{ts} \mathbf{W}_t^\top \mathbf{W}_s) = \frac{1}{n} \mathbb{E}(\mathbf{W}^\top \mathbf{\Omega} \mathbf{W}) \quad (34)$$

Note that we need to add $n^{-1/2}$ inside the variance operator in the first term in order to solve for moment conditions which converge when n grows large (approaches infinity). And then since $\mathbb{E}(u_t | \mathbf{W}_t) = 0$, we have that for all $t = 1, \dots, n$

$$\mathbb{E}(\mathbf{W}_t^\top (\mathbf{y}_t - \mathbf{X}_t \beta)) = 0 \quad (35)$$

These n equations form the theoretical moment conditions. They correspond to the empirical moments on the form:

$$\frac{1}{n} \sum_{t=1}^n w_{ti}^\top (\mathbf{y}_t - \mathbf{X}_t \beta) = \frac{1}{n} w_i^\top (\mathbf{y} - \mathbf{X} \beta) \quad (36)$$

Now let \mathbf{J} be the $l \times k$ full column rank k matrix in (29) such that:

$$\mathbf{J}^\top \mathbf{W}^\top (\mathbf{y} - \mathbf{X} \beta) = 0 \quad (37)$$

This is referred to as the sample moment conditions. Let us assume that the data generating process (DGP) is the one introduced in (31) and that β_0 is the coefficient vector and $\mathbf{\Omega}_0$ is the covariance matrix. We then have that:

$$n^{1/2}(\hat{\beta} - \beta_0) = (n^{-1} \mathbf{J}^\top \mathbf{W}^\top \mathbf{X})^{-1} n^{-1/2} \mathbf{J}^\top \mathbf{W}^\top \mathbf{u} \quad (38)$$

Now using $\mathbb{E}(\mathbf{W}_t^\top (\mathbf{y}_t - \mathbf{X}_t \beta)) = 0$ we get the covariance matrix of the probability limit

$$\begin{aligned} \text{plim}_{n \rightarrow \infty} n^{1/2}(\hat{\beta} - \beta_0) = \\ \left(\text{plim}_{n \rightarrow \infty} \frac{1}{n} \mathbf{J}^\top \mathbf{W}^\top \mathbf{X} \right)^{-1} \left(\text{plim}_{n \rightarrow \infty} \frac{1}{n} \mathbf{J}^\top \mathbf{W}^\top \mathbf{\Omega}_0 \mathbf{W} \mathbf{J} \right) \left(\text{plim}_{n \rightarrow \infty} \frac{1}{n} \mathbf{X}^\top \mathbf{W} \mathbf{J} \right)^{-1} \end{aligned} \quad (39)$$

We now need to find the \mathbf{J} which minimizes this covariance matrix. Following Davidson and Mackinnon (2009, ch. 9.2) we can choose:

$$\mathbf{J} = (\mathbf{W}^\top \boldsymbol{\Omega}_0 \mathbf{W})^{-1} \mathbf{W}^\top \mathbf{X} \quad (40)$$

We then get:

$$\text{plim}_{n \rightarrow \infty} n^{1/2}(\hat{\beta} - \beta_0) = \text{plim}_{n \rightarrow \infty} \left(\frac{1}{n} \mathbf{X}^\top \mathbf{W} (\mathbf{W}^\top \boldsymbol{\Omega}_0 \mathbf{W})^{-1} \mathbf{W}^\top \mathbf{X} \right)^{-1} \quad (41)$$

With the efficient GMM estimator as:

$$\hat{\beta}_{\text{GMM}} = (\mathbf{X}^\top \mathbf{W} (\mathbf{W}^\top \boldsymbol{\Omega}_0 \mathbf{W})^{-1} \mathbf{W}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{W} (\mathbf{W}^\top \boldsymbol{\Omega}_0 \mathbf{W})^{-1} \mathbf{W}^\top \mathbf{y} \quad (42)$$

Note that the GMM estimator reduces to the GIV estimator in (30) if $\boldsymbol{\Omega}_0 = \sigma^2 \mathbf{I}$.

3.3 The Kalman-filter

The method of filtering should be viewed as an art stemming from the seminal work of Gauss (1809) and the famous Gauss Markov Theorem. Gauss shows that it is possible to detect, with increased probability, changes of an unknown event when a new equation is incorporated with some ex-ante computed weights. The Gaussian properties of random variables are the building blocks for the Kalman filter in probabilistic theory. Kalman (1960) proposed a criterion of minimizing the norm of the state vector covariance matrix recursively. The process is to estimate the new state by adding a correction term to the previous state estimate.

In the application that I have in mind the theory suggests that the unobservable component "the desired wage rate", can be accurately extracted from information about the process it follows (or the process it is assumed to follow). To cope with this, I use a Kalman-filter to update the estimates of this unobservable component as we get more information about the process. This means that Kalman-filtration is an iterative process, where we recursively apply additional information to update the information we have about how the component affects the inflation. The Kalman-filter, besides providing mean-squared error forecasts of the endogenous variables and optimal recursive estimates of the unobserved states, is a crucial building block in the prediction error decomposition of the likelihood.

The Kalman-filter assumes that the system can be described by a linear stochastic model with an error term following the normal distribution with zero mean and known variance. The state contains all relevant information relative to the system at a given point in time (Jalles, 2009).

What we are interested in is update on the states $\xi_t, \xi_{t+1}, \xi_{t+2}, \dots$ where we set $\xi_0 = 0$. We want to minimize the noise by filtering the ξ_t . The purpose is then to infer the relevant properties of the ξ 's through ex-ante knowledge about the available observations. We need to employ maximum likelihood to estimate the variance of u_t and \mathbf{v}_t . Then the general formulation of the relevant system is, for $t = 1, 2, \dots$

$$\xi_{t+1} = \mathbf{F}_t \xi_t + \mathbf{B}_t \mathbf{X}_t + \mathbf{v}_{t+1} \quad (43)$$

$$\mathbf{y}_t = \mathbf{H}_t \xi_t + \mathbf{A}_t \mathbf{X}_t + u_t \quad (44)$$

Here, ξ_t is the *state equation* (often referred to as the *transition equation* (Lütkepohl 2005) because it describes the transition between periods) and y_t are observable variables (often referred to as the *observation equation*). \mathbf{X}_t represents the observable inputs or instruments, \mathbf{H}_t is a measurement matrix, \mathbf{A}_t and \mathbf{B}_t are the input matrix of the observation equation and the state equation respectively and \mathbf{F}_t is the transition matrix.

Let the positive and semi-definite covariance matrix of $\mathbf{v}_t = \mathbf{Q}_t$ and that of $u_t = \mathbf{R}_t$ and let \mathbf{v}_t, u_t be Gaussian white noise, and be uncorrelated with the history and each other. Written compactly, we have:

$$\mathbf{v}_{t+1} \sim WN(0, \mathbf{Q}_t) \quad u_t \sim WN(0, \mathbf{R}_t) \quad (45)$$

Note that some of the literature will assume the two error terms to be multivariate normally distributed, which – among other things – implies that uncorrelatedness can be replaced by independence.

$\hat{\xi}_{t|t}$ and $\mathbf{E}_{t|t} = \text{cov}(\xi_t - \hat{\xi}_{t|t})$ are the estimate of the state at time t based on y_0, \dots, y_{t-1} and the error covariance matrix respectively. Some additional nota-

tion useful to describe the recursions of the filter:

$$\hat{\xi}_{t|s} := \mathbb{E}(\hat{\xi}_t | y_1, \dots, y_s) \quad (46)$$

$$\mathbf{y}_{t|s} := \mathbb{E}(\mathbf{y}_t | y_1, \dots, y_s) \quad (47)$$

Following Lütkepohl (2005, ch. 18), the initialization of the iteration will be $\xi_{0|0} = \mu_0$ and $Q_{0|0} = Q_0$. The Kalman-filter algorithm can then be formulated as:

$$\mathbf{Z}_t = \mathbf{y}_t - \mathbf{H}_t \hat{\xi}_{t+1|t} \quad (48)$$

$$\mathbf{S}_t = \mathbf{H}_t \mathbf{E}_{t+1|t} \mathbf{H}_t^\top + \mathbf{R}_t \quad (49)$$

$$\mathbf{K}_t = \mathbf{E}_{t+1|t} \mathbf{H}_t^\top \mathbf{S}_t^{-1} \quad (50)$$

$$\hat{\xi}_{t+1|t+1} = \hat{\xi}_{t+1|t} + \mathbf{K}_t \mathbf{Z}_t \quad (51)$$

$$\mathbf{E}_{t+1|t+1} = (\mathbf{I} - \mathbf{K}_t \beta) \mathbf{E}_{t+1|t} \quad (52)$$

Equation (48) is the innovation, (49) is the innovation covariance, (50) is the Kalman-gain, (51) is the updated state estimate and (52) is the updated state covariance.

For a complete presentation of the Kalman-filter algorithm, see Lütkepohl (2005, ch. 18.3.2). Given initial parameter values, the Kalman-filter can be recursively used to construct the likelihood function and gradient methods can be employed to provide new estimates of the parameters. The two-step method can then be repeated until the gradient or the parameters do not change across iterations.

3.3.1 ARMA(p,q) processes and maximum likelihood estimation

To give an example of how one analytically can apply the Kalman-filter, this section will show how to avoid losing information when estimating an ARMA(p,q)-process using the Kalman-filter. For general multivariate ARMA(p,q) processes, we can use a Kalman-filter to evaluate the successive contributions to the loglikelihood for given parameter values. Thus, the Kalman-filter can serve as the basis of an algorithm for maximizing loglikelihood. Let an ARMAX(p,q) model take the form:

$$y_t = \mathbf{X}_t \beta + u_t \quad u_t \sim ARMA(p, q) \quad \mathbb{E}(u_t) = 0 \quad (53)$$

Then the easiest estimation method is just to drop the first p observations and use non-linear least squares to estimate the non-linear regression model:

$$y_t = \mathbf{X}_t\beta + \sum_{i=1}^p \rho_i(y_{t-i} - \mathbf{X}_{t-i}\beta) + \varepsilon_t \quad (54)$$

Now, we do not want to lose the information in the first p observations if we can avoid it. Thus, under the assumption that u_t is stationary and ε_t is white noise, we can use maximum likelihood estimation. When we assume that ε_t are normally distributed, it follows directly that the $ARMA(p, q)$ process in the error terms u_t are normally distributed, and then also the dependent variable y_t conditional on the explanatory variables. Let y denote the n -dimensional vector of which the elements are y_1, \dots, y_n . Then the expectation of y is $\mathbf{X}\beta$ and $\mathbf{\Omega}$ is the autocovariance matrix of the vector y :

$$\mathbf{\Omega} = \begin{bmatrix} v_0 & v_1 & v_2 & \cdots & v_{n-1} \\ v_1 & v_0 & v_1 & \cdots & v_{n-2} \\ v_2 & v_1 & v_0 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ v_{n-1} & v_{n-2} & v_{n-3} & \cdots & v_0 \end{bmatrix} \quad (55)$$

where v_i is the stationary covariance of u_t and u_{t-i} and v_0 is the stationary variance of u_t . If we now use the multivariate normal density function, we get that the log of the joint density of the observed sample is:

$$-\frac{n}{2} \log 2\pi - \frac{1}{2} \log |\mathbf{\Omega}| - \frac{1}{2} (y - \mathbf{X}\beta)^\top \mathbf{\Omega}^{-1} (y - \mathbf{X}\beta) \quad (56)$$

The problem of estimating this model is that $\mathbf{\Omega}$ is a $n \times n$ matrix. We can solve this by finding an upper triangular matrix $\mathbf{\Psi}$ such that $\mathbf{\Psi}\mathbf{\Psi}^\top = \mathbf{\Omega}^{-1}$. It is then possible to solve a system where all contributions are additive.

4 Empirical results

This section will use the proposed econometric methodology to the framework presented in Section 2. First, I describe how the empirical work has been conducted, and then the results are presented.

4.1 Data description

There are a number of choices to be made when modeling the hybrid Wage-NKPC, and this section will thoroughly cover which choices have been made and why. The system will be estimated and calibrated based on annual Norwegian data from 1970-2014 available from Statistics Norway. In this section, I present how the annual data has been constructed. w_t is estimated using log of wages in millions of kroner divided by man-year, full-time equivalents (1000 man-years) so that:

$$w_t = \ln \left(\frac{\text{Wages in 1000 kroner}}{\text{Man-years}} \right) \quad (57)$$

Already here, a choice has been made. I chose to use man-years (full time equivalents) instead of the actual number of employed persons, because the man-year variable will calculate part-time workers into full-time equivalents. Thus, using the actual number of employed persons would possibly lead to an underestimation of the wage because we would divide by a too large number, seeing as part-time workers would be counted equal as full-time workers. The wage-inflation variable π_t^w is then the one-period difference of w_t , π_{t-1}^w is the one-period lag of π_t^w , π_{t+1}^w is the one-period lead of π_t^w which will be estimated with instrumental variables, c_t is estimated using log of consumption in millions of kroner (in 2005 price-levels), divided by man-year, full-time equivalents (1000 man-years) so that:

$$c_t = \ln \left(\frac{\text{Consumption in 1000 kroner}}{\text{Man-years}} \right) \quad (58)$$

Δc_t is the one-period difference of c_t , l_t is measured as log of man-year, full time equivalents (1000 man-years), y_t is log of gross domestic product (in 2005-price levels) divided by man-year, full time equivalents (1000 man-years), so that:

$$y_t = \ln \left(\frac{\text{GDP in 1000 kroner}}{\text{Man-years}} \right) \quad (59)$$

Finally mc_t is measured as log of wage costs divided by gross domestic product in current prices in millions of kroner. This stems from a simplifying assumption of Cobb-Douglas technology where the marginal costs are the unit labor cost, so that:

$$mc_t = \ln \left(\frac{\text{Wage bill}}{\text{GDP}} \right) \quad (60)$$

This is the log of the wage share of income (or equivalently the labor share of income) and is a standard proxy for real marginal costs which is unobservable. This is of course a simplification and a choice made to derive a tractable and interpretable measure of the marginal costs. Both the numerator and denominator are in current prices. The specification presented here uses "man-year" as denominator value. As a test of robustness, I will also present some results using "the number of hours worked" as denominator.

Figure 1 shows the time series plot for all the variables used in the analysis. Panel a is the time series for the (log of) inflation, panel b is the time series for the (log of) consumption, panel c is the first difference of panel b, panel d is the time series for the (log of) productivity, and panel e is the time series for the (log of) marginal costs. All variables are following reasonable patterns over time as Figure 1 displays.

4.2 Application of GIVE to the Hybrid Wage-NKPC

When the expected rate of wage inflation is substituted by the actual rate of wage growth in period $t + 1$, we get an endogeneity problem since the lead-in-wage inflation is correlated with the error term in the NKPC. Hence, it is necessary to use an instrumental variable approach. The procedure is based on the orthogonality conditions that evolve from the underlying theory as introduced in chapter 3.

When applying the general IV estimator to the hybrid-NKPC, we construct the imported inflation using annual data on import prices (constructed by taking the ratio of the import in current prices to import in 2005 price levels, taking the logarithm of this and one-period difference it). We further use the first difference of the log of the productivity level in the manufacturing sector as an instrument. From business cycle theory, we know that this indicator is a leading indicator in the cycle, see for instance Acemoglu (2009) or Roth (1986). We take the first difference to get rid of non-stationary trends. We also use the logarithm of the

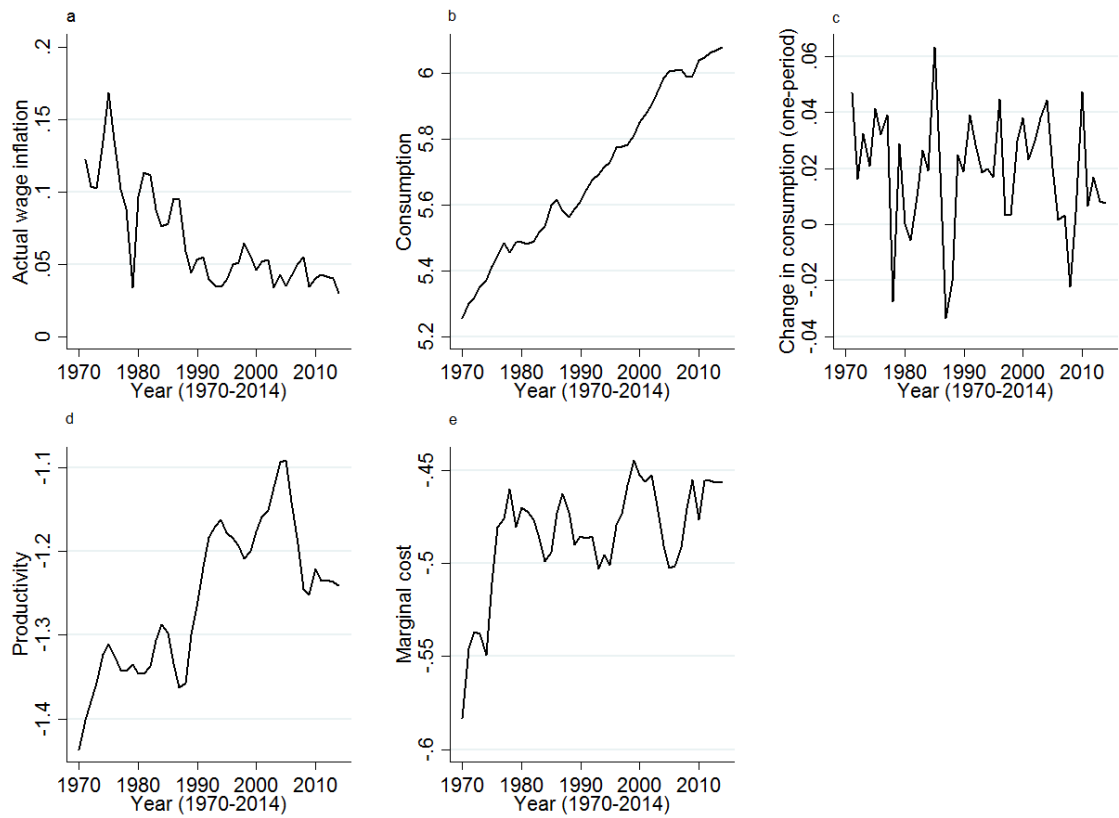


Figure 1: Plot of all the variables used in the analysis, constructed as described above.

unemployment which has a correlation of 0.7578 with the lead inflation. Following Galí and Gertler (1999), we add two lags of inflation as instruments. These instruments work as the IV estimators for the lead inflation (or the expected future inflation). We "naively" skip the Kalman-filter for now and will use the results from this approach as a benchmark for both the GMM and when applying Kalman-filter.

In the code, we let $A_t = (y_t - l_t)$ denote the measure of productivity (in logs). This is also how productivity in the manufacturing sector is constructed.

Figure 2 shows clear indications of imported inflation "leading the inflation", so that imported inflation today should be a good indicator of inflation tomorrow. The correlation between imported inflation and lead inflation was estimated to be 0.7221. Figure 3 show the same indication for the productivity in the manufacturing also leading the inflation and thus could be a good proxy for the inflation tomorrow.

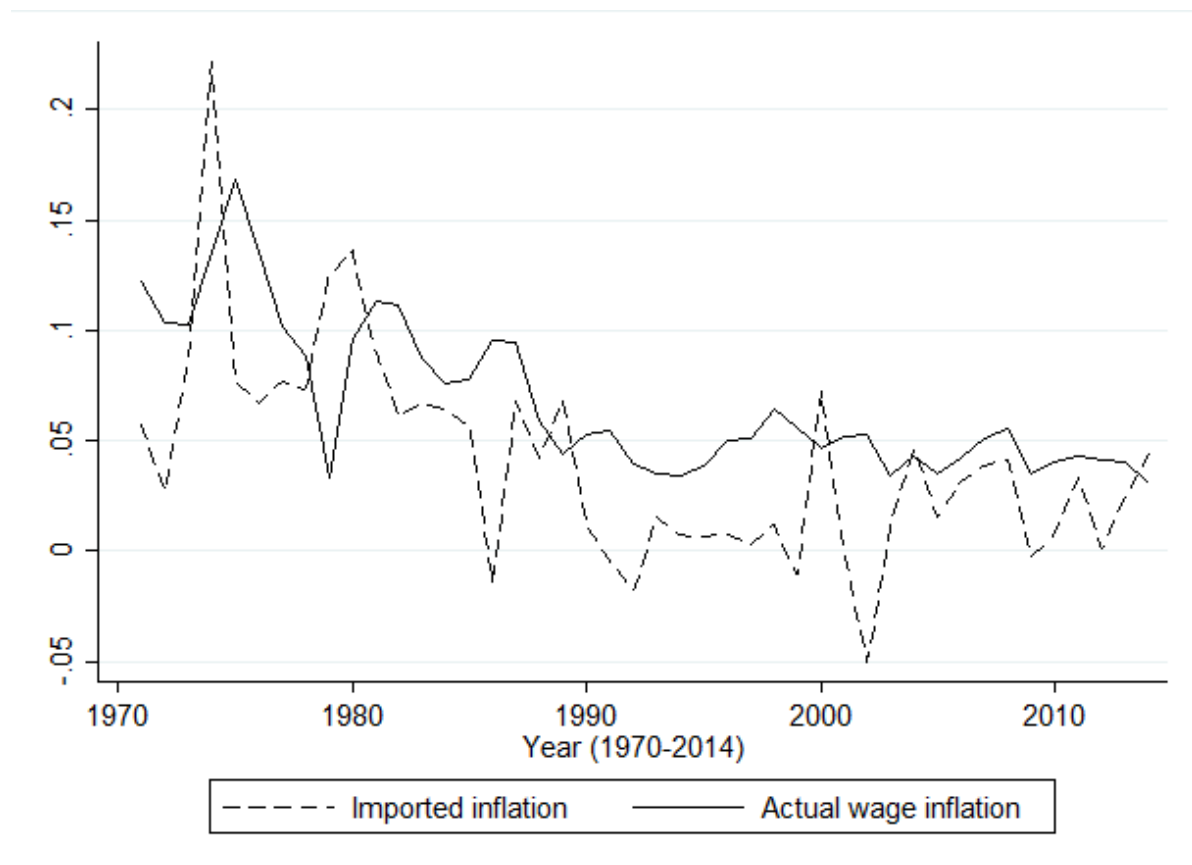


Figure 2: Time series plot of imported inflation and actual inflation.

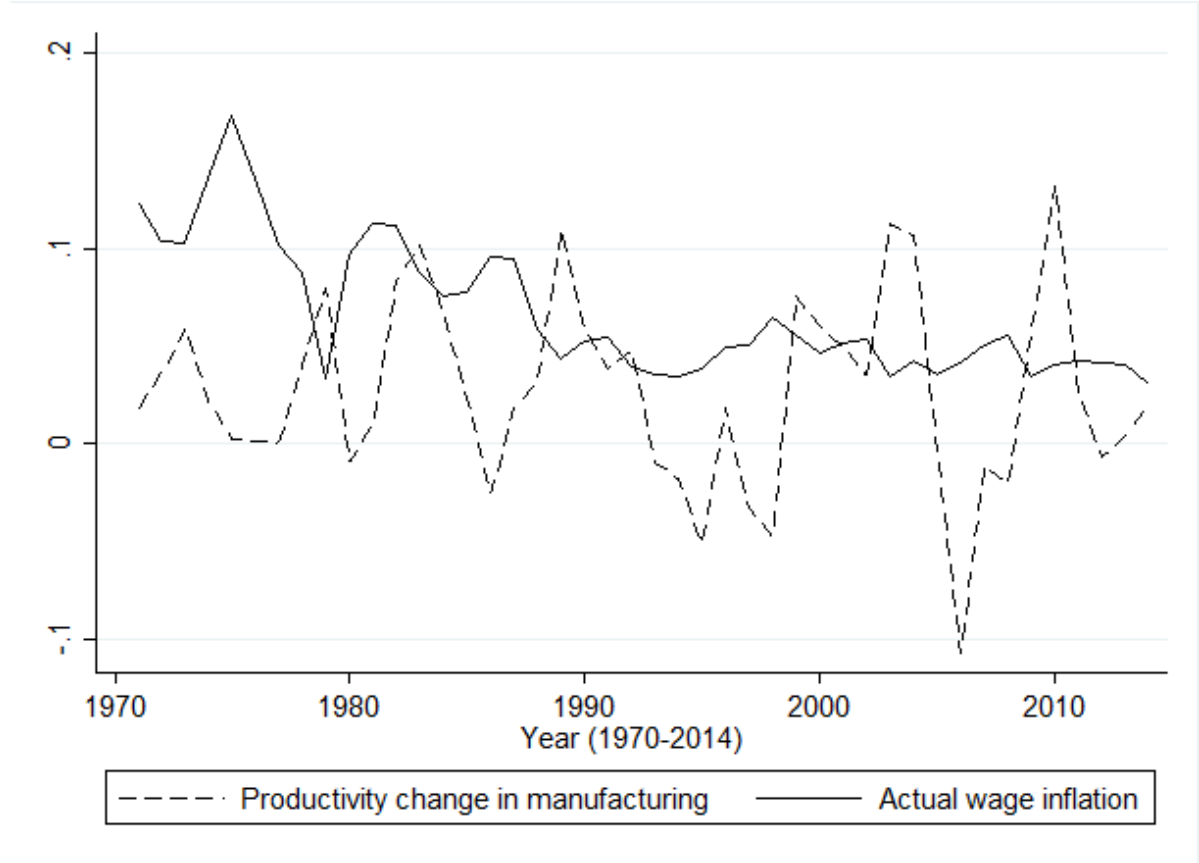


Figure 3: Time series plot of the first-difference of the log of productivity in the manufacturing sector and the actual inflation.

4.3 Application of GMM to the Hybrid Wage-NKPC

The application of GMM follows much of the procedure of GIVE, except that we specify the GMM-procedure in the software (Stata SE13.1). We use the same instrumental variables for numerically consistent and comparable results. The only real advantage of using the GMM procedure rather than the two-stage least squares (2SLS) in the general IV is to correct for potential autocorrelated residuals. Following Galí and Gertler (1999), we know that under rational expectations, the error in the forecast of π_{t+1}^w is uncorrelated with information dated t (and earlier), which gives us that:

$$\mathbb{E}_t \{ (\pi_t^w - \alpha_b \pi_{t-1}^w - \alpha_f \pi_{t+1}^w - \tau(w_t^* - w_t) - \varepsilon_{\pi t}^w) z_t \} = 0 \quad (61)$$

Where z_t is a vector of variables dated t and earlier. This orthogonality property is the basis for the GMM-procedure.

Note that past empirical work, such as Hornstein (2008) argues that lagged inflation tends to be a good forecast of expected future inflation and that it is hard to improve on that forecast. This will suggest that the instruments in both GIVE and GMM will be quite weak. To cope with this issue, the Sargan-Hansen-test of overidentifying restrictions, the Kleibergen-Paap LM-test of underidentifying restrictions and the Cragg-Donald Wald F-test of weak identification will be reported for the GIVE and GMM. The Kleibergen-Paap LM test was proposed by Kleibergen and Paap (2006) and ultimately tests the rank of a matrix, which leads to testing the underidentification of the model. The Cragg-Donald Wald F test was proposed by Cragg and Donald (1993) and explores moment specifications for the identifiability of parameters estimable by instrumental variables. It thus tests for weak identification of the model. The full reports in the following sections will be for the Norwegian mainland economy. Reports for the total Norwegian economy, the public sector and the manufacturing sector will be added to Appendix B.

The wage equations have been augmented by dummies for the years 1988 and 1989. These were extremely difficult years for the Norwegian economy, and among the measures taken was the Norwegian parliament stipulated the wage growth for 1988, in the form of a "wage law". The parliament passed a wage-law also in 1989, hence these two years can be characterized by supercentralized wage setting, see Stokke (1997). Of course, the theoretical model does not incorporate highly centralized wage negotiation, but assumes instead independent wage setting by the households and respondent ability to pay wages by firms. Including the years 1988 and 1989 in the sample would therefore mean that the evidence is tilted against the model. Econometrically, they would also represent outliers that can adversely affect the properties of estimators. The dummies are "partialled-out" because adding them would result in a robust covariance matrix with less than full rank, and thus not a valid Sargan-test and uninterpretable standard errors. By the Frisch-Waugh-Lovell theorem (Frisch and Waugh, 1933) the coefficients for the remaining exogenous regressors are the same as those that would be obtained if the variables were not partialled out, hence no econometric value is lost in partialling out those dummies.

Table 1: GIVE and GMM on the Wage-NKPC using annual data. Man-year denominator, full sample

	GIVE (1970-2014)	GMM (1970-2014)
$\mathbb{E}\pi_{t+1}$	0.6186*** (0.1232)	0.6583*** (0.1120)
π_{t-1}	0.5016*** (0.1046)	0.3795*** (0.0755)
c	-0.0347 (0.0327)	-0.0601** (0.0273)
Δc	-0.2345** (0.0938)	-0.1751*** (0.0673)
l	0.0420 (0.0351)	0.0652** (0.0295)
A	0.0636 (0.0634)	0.0537 (0.0494)
mc	0.0837 (0.0783)	0.1590** (0.0685)
N	40	40
R^2	0.968	0.966
adj. R^2	0.959	0.956
Sargan J-statistic	5.915	5.915
Chi-sq(4) P-val	0.2056	0.2056

Standard errors in parentheses

Instruments used to instrument for lead inflation: log of imported inflation, log of unemployment rate, log of productivity in the manufacturing sector and two lags of inflation.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Arellano-Bond test of autocorrelation

GIVE (1970-2014)		
Arellano-Bond test for AR(1):	$z = -1.79$	$\Pr > z = 0.0730$
Arellano-Bond test for AR(2):	$z = 1.26$	$\Pr > z = 0.2079$
Arellano-Bond test for AR(3):	$z = -0.00$	$\Pr > z = 0.9967$
Arellano-Bond test for AR(4):	$z = -0.94$	$\Pr > z = 0.3465$
Arellano-Bond test for AR(5):	$z = 1.08$	$\Pr > z = 0.2783$
GMM (1970-2014)		
Arellano-Bond test for AR(1):	$z = -0.93$	$\Pr > z = 0.3507$
Arellano-Bond test for AR(2):	$z = 0.91$	$\Pr > z = 0.3632$
Arellano-Bond test for AR(3):	$z = -0.67$	$\Pr > z = 0.5010$
Arellano-Bond test for AR(4):	$z = -1.00$	$\Pr > z = 0.3189$
Arellano-Bond test for AR(5):	$z = 0.87$	$\Pr > z = 0.3821$
The null hypothesis is		
no autocorrelation between the residuals.		

4.4 Results of the GIVE approach

The first thing to notice in Table 1 is that we get coefficient estimates of the lead- and lagged inflation which are significant at any conventional level of significance (even without robust standard errors, but this output is omitted). These coefficients sum to 1.1201 which is fairly close to the homogeneity restriction 1. Consistent with theory is also the fact that the forward-term is larger than the backward-term as mentioned earlier. An F-test of the joint probability that the two coefficients of the expected future inflation and the lagged inflation sum to 1 cannot be rejected at any conventional level of significance (p-value = 0.4125). The joint test of significance of the forcing variables returns a p-value of 0.0336, which means that we can reject the null of joint insignificance at a 5 percent level, but not at a 1 percent level. We can thus conclude, if being liberal in the choice of significance level, that the coefficients of the forcing variables are jointly significant

We can then see that none of the other explanatory variables are significant at conventional significance levels except the change in consumption; however they are close to significance. This is a puzzle, because all the forcing variables should be significant using GIVE. This can be an indication of highly (negatively) autocorrelated residuals. The Sargan-Hansen J-statistic (the $\chi^2_J(4)$ -test) is 5.915 with a p-value of 0.2056. This suggests that the instruments included are relevant, be-

cause we cannot reject the null hypothesis of valid over-identification. We also see from the Kleibergen-Paap LM-statistic that we can reject the null hypothesis of underidentification at any conventional level of significance⁴.

We also present the Arellano-Bond test for autocorrelation in Table 2. This test was proposed by Arellano and Bond (1991) and was originally proposed as a test for a particular GMM dynamic panel data estimator, but has been proven general in its applicability, see for instance Roodman (2006) for an overview. It applies to GMM estimators in general, and since GIVE is a special case of GMM it also applies to GIVE (as well as 2SLS and OLS). It is robust to various patterns of error covariance. We test for autocorrelation in the first 5 lags of the residuals. The null hypothesis of the test is no autocorrelation, so if we cannot reject the null, it indicates that we have no autocorrelation. In the GIVE-results, we can reject the null of no autocorrelation for the first lag at a 10 percent level, but not at a 5 percent level. Closer inspection shows that the autocorrelation is negative if it is present, which would cause us to overestimate the standard errors. This indicates that if anything, we have too little significance in the GIVE results.

4.5 Results of the GMM approach

The coefficients on the lead- and lagged inflation are both significant at any conventional significance level also for the GMM approach (see Table 1). These coefficients now sum to 1.0378. The forward-term is still larger compared to the backward-term, which is consistent with the theory. We can note that the robust standard errors are changed in magnitude. This may be a sign of autocorrelation in the model, which suggests that we should trust the GMM. Note that the estimates in the GMM approach are efficient for arbitrary heteroskedasticity, while the GIVE-estimates are efficient for homoskedasticity only. In the GMM approach, all forcing variables except the productivity are significant down to a 5 percent level, but only the change in consumption is significant at a 1 percent level.

All signs in the GMM results are the same as with the GIVE approach.

The Sargan Hansen J-test again suggests that the null is not rejected, so we can conclude that the overidentifying restrictions are not invalid.

⁴For the full table of results, see Appendix A

An F-test of the joint probability that the two coefficients of the expected future inflation and the lagged inflation sum to 1 cannot be rejected at any conventional level of significance (p-value = 0.7557). The joint test of significance of the forcing variables returns a p-value of 0.0013 (and a $\chi^2_J(5)$ -statistic of 19.88), which means that we can reject the null of joint insignificance at any conventional level of significance. This suggests that the GMM procedure performs better due to the optimization of the residual covariance matrix. With the GMM procedure, the forcing variables are at least jointly significant down to conventional (but possibly conservative) levels⁵.

The Arellano-Bond test in Table 2 shows that we cannot reject the null of no autocorrelation at any conventional level of significance for the first 5 lags of the residuals.

4.5.1 Actual versus fundamental inflation

Equation (25) shows the solution for the actual inflation implied by the model. As a way to assess the model's goodness-of-fit, we follow Galí and Gertler (1999) and plot this actual inflation versus the fundamental inflation estimated by the model. The model-based estimation is termed "fundamental" inflation, since this is analogous to how Galí and Gertler (1999) presents their model estimations.

In order to solve the estimated version of (25), we need to find parameter values for the AR(1)-coefficients of the process of all the forcing variables in (12)-(16). The results are reported in Table 9. We get reasonable AR(1)-coefficients, all significant at any level of significance, for all the variables. All the other components needed to solve (25), namely the estimated coefficients, are taken from Table 1.

The plot⁶ in Figure 4 shows that overall fundamental inflation is much more volatile than actual inflation. We have some trouble with the sum of the lead and lagged coefficients summing to more than 1, which results in the roots in (21) both being smaller than 1 in magnitude; $\tilde{\alpha}_{b,1} = 0.739$ and $\tilde{\alpha}_{b,2} = 0.780$. As we can see from (25) this means that the contribution to the fundamental inflation from

⁵For the full table of results, see Appendix A

⁶In the discussion, we have seen that GMM generally performs better than GIVE due to the more general covariance matrix of the residuals. Thus, only the plot for GMM is reported.

each of the forcing variables is scaled up. Thus, the implied fundamental inflation is unable to track actual wage inflation well in this approach. The correlation between fundamental inflation and actual inflation in this approach is only 0.1810 which is quite low.

Table 3: AR(1)-processes of the forcing variables

	AR(1)-coefficient
c	0.986*** (0.013)
l	0.9*** (0.0188)
A	0.909*** (0.0423)
mc	0.743*** (0.06894)
Standard errors in parentheses	
* p<0.10, ** p<0.05, *** p<0.01	

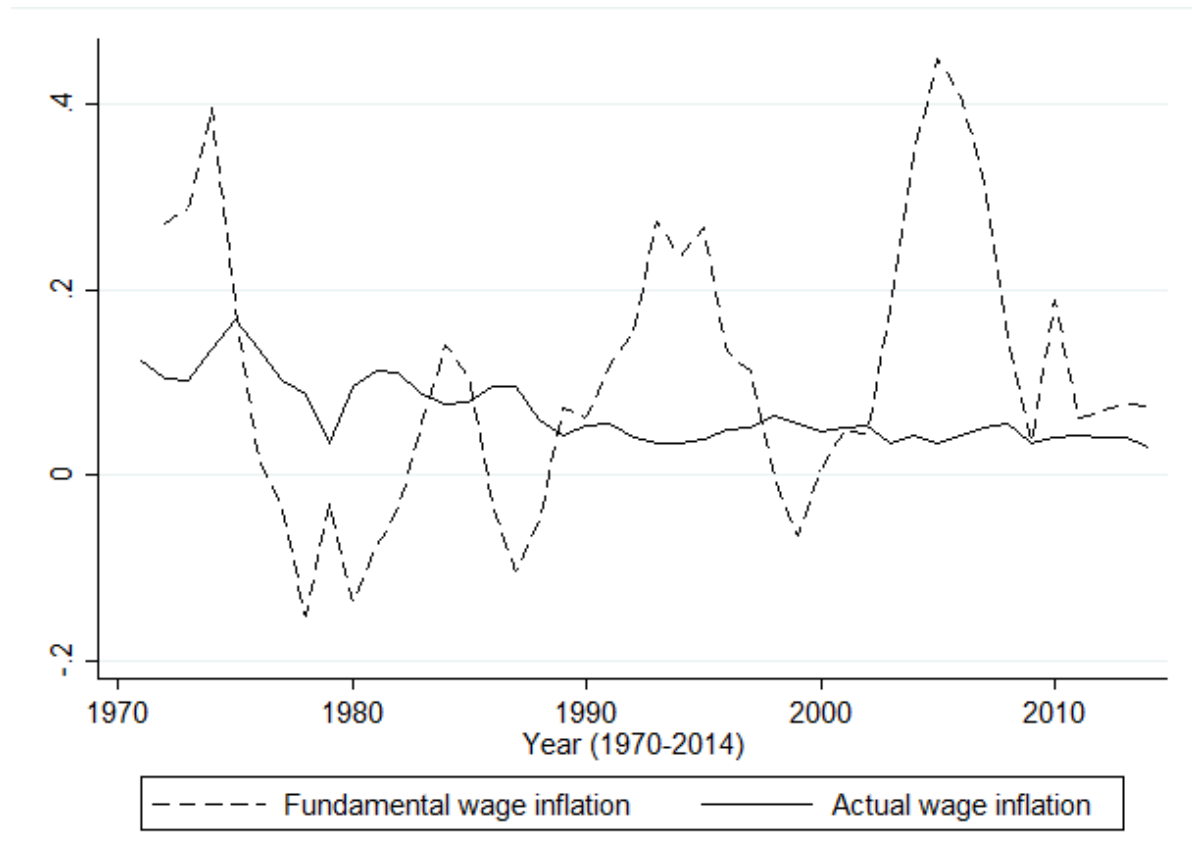


Figure 4: Inflation: Actual vs. Fundamental, GMM method predicting fundamental inflation.

4.6 Discussion of the forcing variables used in GIVE and GMM

The annual data gives coefficients on the forcing variables which contradict the theory to some extent, both using GIVE and GMM. The coefficients on two of the components of the pressure indicator are negative, even though theory suggests they be positive. From the estimation, a percentage increase in consumption or a percentage change in consumption significantly decreases wage inflation. This has no immediately obvious interpretation; theory suggests that higher consumption implies a higher demand and thus a higher demand for labor to increase supply of the final good.

The two components in the ability to pay wages indicator are both positive, contradicting the theoretical negative sign on the marginal costs, but verifying the theoretical positive sign on productivity. This indicates that a percentage increase in productivity or the marginal costs will increase wage inflation.

Productivity is highly linked to output, so that if output is high it may be that the reason for that is an increasing demand which would incentivize firms to hire more workers and produce more to increase the surplus. This would correspond to an increase in the firms' "ability to pay wages". If that is the case, then the positive sign can be justified. Then there is also the counter argument that a higher productivity means the firm needs to hire fewer workers to produce the same amount, which would put a downward pressure on wages. It seems from this data that the former effect dominates the latter, which is consistent with theory. However, note that the coefficient on productivity is insignificant at conventional levels of significance in both GIVE and GMM.

The marginal costs are closely linked to the firms' profits. If marginal costs increase, the firms will respond by increasing the price of the final good. This, then, drives up wage inflation because wage-earners want to catch up to the rising prices to avoid a downfall in the real wages. This effect seems to dominate, contradicting the theory. The counter argument is that a higher marginal cost will dis-incentivize the firms to hire more workers and rather let some workers go. This puts a downward pressure on wages, because workers will accept a lower wage in order to be employed rather than risk being unemployed and thus receive zero

payment. Seemingly, this estimation provides an argument for marginal cost to have a different interpretation than being a component in the firms' ability to pay wages. It seems as if high marginal costs will increase prices and in turn increase wage pressure, a contradicting effect to the one proposed by Brubakk and Sveen (1/2009), which would imply that the effect of firms letting workers go and decrease production is a dominating effect.

The coefficient on the employment level is consistent with the theory; positive and significant for GMM, and positive and insignificant for GIVE. This suggests that a high level of employment puts pressure on the labor market and gives negotiation power to the households. Since the employment level is high, the workers who are currently unemployed possess a negotiation power because more firms wish to employ their specialized labor supply and they compete with fewer households in providing it.

4.6.1 Robustness checks

To test the robustness of these results, a number of different analyses can be conducted. First and foremost, we should test whether the forward-dominance is a result of using two lags of inflation as instruments. The forward-dominance could be a result of not allowing for sufficient dependency on lagged inflation in the structural equation. We address this by adding two additional lags to the right hand side of the structural equation and see if they have any predictive power beyond the signaling power they have on the expected future inflation.

The results are reported in Table 4. We can see that none of the coefficients on the forcing variables or the lead and one-period-lagged inflation change dramatically. In addition, the joint significance of the two additional lags is rejected for both procedures at any conventional level of significance (p-values 0.7394 and 0.8156 respectively). It seems as if the model accounts for inflation inertia without the need of reliance on arbitrary lags. The Arellano-Bond test is reported in Table 5 and indicates the same pattern as in the model without additional lags included as explanatory variables.

Another test of the specification is whether the model is robust to a different denominator (which has been man-years, full time equivalents). The model has been re-estimated using man-hours (millions of man-hours). The results are re-

ported in Appendix A. Tables B.5 and B.6 show the results. The significance of some of the forcing variables in the GMM approach disappears in these models, however in the GIVE approach some variables are closer to significance with the new specification of the denominator. Some of the coefficients are not robust to the use of this alternative denominator, for instance labor supply. However, that coefficient was not significant when using man-years as denominator value and is still insignificant for both GIVE and GMM. In sum, few conclusions can be drawn from this. However, we do note that the coefficients on the lead and lagged inflation are still significantly different from 0 and we still have forward dominance. The magnitude of these coefficients are not dramatically changed. Thus, this robustness check seems to be in favor of the model.

The third test of robustness is to estimate different subsamples. Table 6 shows the results of using GMM on the sample from 1970-1989 (adding dummies for 1988 and 1989) and 1990-2014 respectively. Evidently, the results are not robust to the subsample test at all. All coefficients dramatically change, which at first sight undermine the previous results. This test, though, is of limited importance because of the number of observations compared to the number of explanatory variables in the first-stage regression. There are only 16 observations in the first sample and 24 in the second. In the first-stage regression of the subsamples, there are 11 explanatory variables. Hence, the estimations are likely to be ridden with considerable error, and we can draw few statistically significant concluding remarks from this, if any at all.

Table 4: GIVE and GMM on the Wage-NKPC using annual data, adding additional lags as explanatory variables. Denominator man-year, full sample

	GIVE (1970-2014)	GMM (1970-2014)
$\mathbb{E}\pi_{t+1}$	0.5830*** (0.1336)	0.6746*** (0.1263)
π_{t-1}	0.5117*** (0.1451)	0.3249*** (0.1186)
c	-0.0319 (0.0339)	-0.0547* (0.0300)
Δc	-0.2047** (0.1001)	-0.1811* (0.0958)
l	0.0316 (0.0378)	0.0579* (0.0349)
A	0.0235 (0.0604)	0.0380 (0.0595)
mc	0.0392 (0.1088)	0.1439 (0.0939)
π_{t-2}	0.0033 (0.1166)	0.0634 (0.1127)
π_{t-3}	-0.1113 (0.1564)	-0.0671 (0.1241)
N	40	40
R^2	0.969	0.966
adj. R^2	0.957	0.953
Sargan J-statistic	5.538	5.538
Chi-sq(4) P-val	0.0627	0.0627

Standard errors in parentheses

Instruments used to instrument for lead inflation: log of imported inflation, log of unemployment rate, log of productivity in the manufacturing sector

* p<0.10, ** p<0.05, *** p<0.01

Table 5: Arellano-Bond test of autocorrelation, with additional lags of inflation as explanatory variables

GIVE (1970-2014)		
Arellano-Bond test for AR(1):	$z = -1.79$	$\Pr > z = 0.0742$
Arellano-Bond test for AR(2):	$z = 1.11$	$\Pr > z = 0.2654$
Arellano-Bond test for AR(3):	$z = 1.19$	$\Pr > z = 0.2354$
Arellano-Bond test for AR(4):	$z = -1.04$	$\Pr > z = 0.2994$
Arellano-Bond test for AR(5):	$z = 0.87$	$\Pr > z = 0.3848$

GMM (1970-2014)		
Arellano-Bond test for AR(1):	$z = -0.93$	$\Pr > z = 0.3507$
Arellano-Bond test for AR(2):	$z = 0.91$	$\Pr > z = 0.3632$
Arellano-Bond test for AR(3):	$z = -0.67$	$\Pr > z = 0.5010$
Arellano-Bond test for AR(4):	$z = -1.00$	$\Pr > z = 0.3189$
Arellano-Bond test for AR(5):	$z = 0.87$	$\Pr > z = 0.3821$

| The null hypothesis is | | |
| no autocorrelation between the residuals. | | |

Table 6: GMM on the Wage-NKPC using annual data, robustness checks. Denominator man-year, subsamples sample

	GMM (1990-2014)	GMM (1970-1989)
$\mathbb{E}\pi_{t+1}$	1.1893*** (0.3360)	1.0829*** (0.3534)
π_{t-1}	0.2829 (0.2977)	0.5876*** (0.1925)
c	0.0260 (0.0623)	0.0265 (0.1670)
Δc	0.0418 (0.0875)	-0.4965* (0.2597)
l	-0.0317 (0.0752)	0.0625 (0.1600)
A	-0.0692 (0.1068)	0.2898 (0.3476)
mc	0.0327 (0.1417)	0.5680 (0.3485)
N	24	16
R^2	0.964	0.958
adj. R^2	0.949	0.941
Sargan J-statistic	5.538	5.538
Chi-sq(4) P-val	0.0627	0.0627

Standard errors in parentheses

Instruments used to instrument for lead inflation: log of imported inflation, log of unemployment rate, log of productivity in the manufacturing sector and two lags of inflation.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

4.6.2 Unit root test of stationarity

We should test for unit root stationarity in (10) by inserting for the empirical coefficients. Doing this gives:

Table 7: Unit root test of stationarity

	α_f	α_b	v	ρ	Roots
GIVE	0.6186	0.5016	0	0.842	$\{0.842, 0.808 + 0.397i, 0.808 - 0.397i\}$
GMM	0.6583	0.3795	0	0.842	$\{0.874, 0.743 + 0.0516i, 0.743 - 0.0516i\}$

As we can see, we get only one non-imaginary root for both the methods. Thus, we need to calculate the module of the two imaginary roots for each method. The formula to calculate the module of an imaginary root is if the root is on the form $a - bi$, then the module is $m = \sqrt{a^2 + b^2}$. The modules for GIVE are thus $m_1 = \sqrt{0.808^2 - 0.397^2} = 0.495, m_2 = \sqrt{0.808^2 + 0.397^2} = 0.810$. The modules for GMM are: $m_3 = \sqrt{0.743^2 - 0.0516^2} = 0.549, m_4 = \sqrt{0.743^2 + 0.0516^2} = 0.555$. Since none of the modules are larger than 1 and the root is not 1, the system has a stationary solution for both GIVE and GMM.

4.7 Restricting the sum of coefficients on lead and lagged inflation

Since the results show that the sum of the coefficients exceeds 1 for both GIVE and GMM, and we know that there are several properties of the model which change if the sum is below 1, this section will impose the restriction that the sum of the coefficients on the lead and lagged inflation is $\alpha_b + \alpha_f = 0.99$. This casts the model into the following form:

$$\pi_t^w - 0.99\pi_{t-1}^w = \alpha_f(\mathbb{E}_t\pi_{t+1}^w - \pi_{t-1}^w) + \tau[\phi c_t + \psi\Delta c_t + \gamma l_t - \kappa(y_t - l_t) - \zeta mc_t] + \varepsilon_{\pi t}^w \quad (62)$$

Which means that we have to reformulate the specification to estimate with a transformed left-hand-side variable and a transformed right-hand-side variable. The results are reported in Table 8. We can see that GMM still exhibits forward-dominance, however GIVE does not. Hence the precise estimate of the fraction of backward-looking wage-setters is sensitive to the use of this restriction, at least in GIVE. The Sargan-Hansen J-test returns a p-value of 0.5297 which means that

the instruments included are still relevant.

Now addressing whether this method provides better estimates of tracking actual inflation, the fundamental inflation is calculated and fitted. With reference to Nymoen et al. (2012), the restriction we have imposed must mean that the second root in (21) must be greater than 1 in magnitude. By re-calculating the roots, we get that $\tilde{\alpha}_{b,1} = 0.732$ and $\tilde{\alpha}_{b,2} = 1.067$. The resulting fundamental versus actual inflation plot is displayed in Figure 5. The correlation between fundamental inflation and actual inflation is 0.7667 which is quite high. The figure shows, however, that the correlation may stem from fundamental inflation overestimating and underestimating in almost equal portions. Fundamental inflation is able to track actual inflation at least to some extent, and seems to do well on average over time.

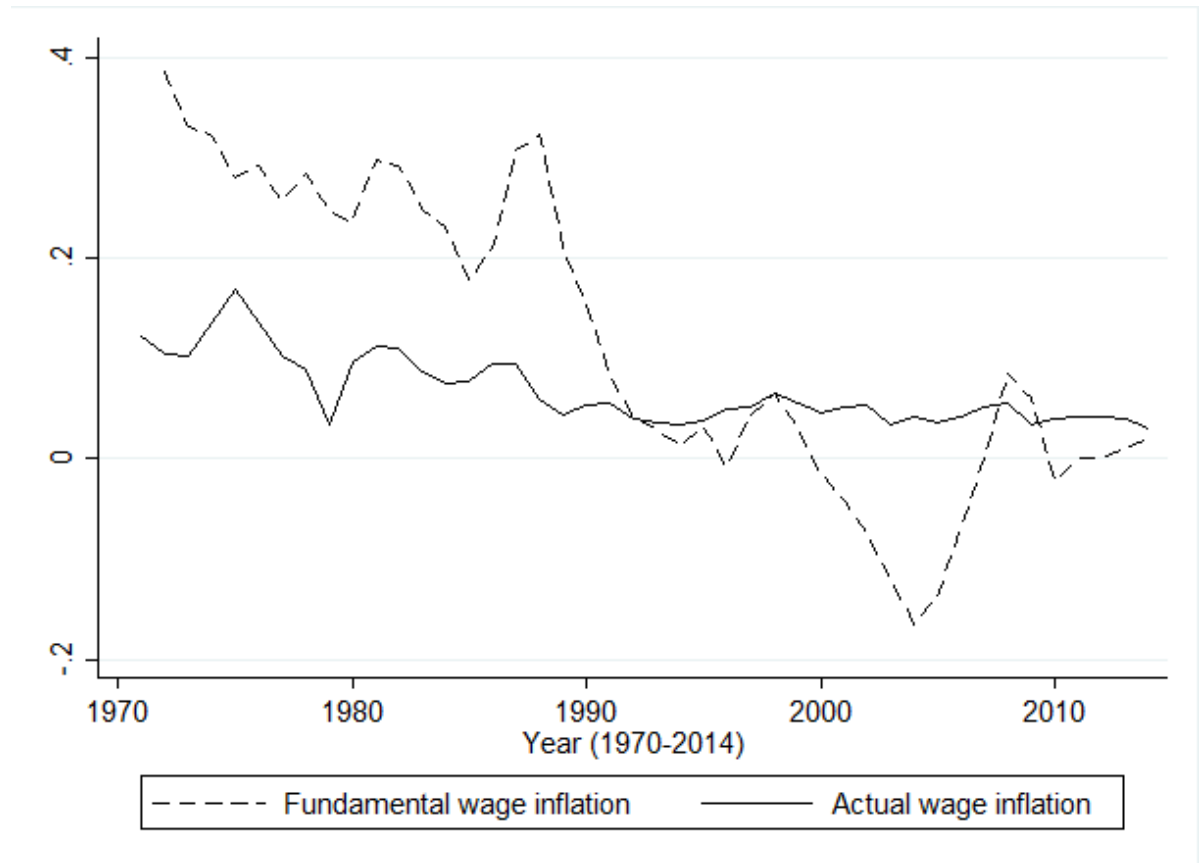


Figure 5: Inflation: Actual vs. Fundamental, GMM method predicting fundamental inflation. Restricted coefficients.

Table 8: GMM and GIVE on the Wage-NKPC using annual data, restricted coefficients. Denominator man-year, full sample

	GIVE (1970-2014)	GMM (1970-2014)
$\mathbb{E}\pi_{t+1}$	0.4852*** (0.1048)	0.5559*** (0.0725)
π_{t-1}	0.5048*** (0.1048)	0.4341*** (0.0725)
c	-0.0238 (0.0317)	-0.0431 (0.0279)
Δc	-0.2019** (0.0961)	-0.2295*** (0.0585)
l	0.0227 (0.0344)	0.0466 (0.0295)
A	0.0194 (0.0539)	0.0317 (0.0412)
mc	0.0084 (0.0937)	0.1203* (0.0663)
N	40	40
R^2	0.543	0.533
adj. R^2	0.429	0.416
Sargan J-statistic	3.170	3.170
Chi-sq(4) P-val	0.5297	0.5297

Standard errors in parentheses

Instruments used to instrument for lead inflation: log of imported inflation, log of unemployment rate, log of productivity in the manufacturing sector and two lags of inflation.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

4.8 Application of the Kalman-filter to "desired wage rate"

Let us first apply the Kalman-filter theoretically to the Wage-NKPC. The model contains two observation equations, namely the hybrid-NKPC and decomposition of the forcing variable into a stationary trend and the desired wage (which is a cyclical non-stationary component). The two unobservable components are the trend and the desired wage. We define the desired wage as an AR(1) process and the trend as a random walk with drift. This means that we can write the state space representation of the (simple) Wage-NKPC as follows, where the periods are rewritten and restructured so we get a two-period lagged form instead of one lag and one lead:

$$\pi_t^w = \frac{1}{\alpha_f} \pi_{t-1}^w - \frac{\alpha_b}{\alpha_f} \pi_{t-2}^w - \frac{\tau}{\alpha_f} \Phi_t - \frac{1}{\alpha_f} \varepsilon_{\pi t}^w \quad (63)$$

$$\Phi_t = w_t^* - w_t + \hat{w}_t \quad (64)$$

The second equation is the forcing variable, and the *state equation* in this model. The first equation is the *observation equation*, which we partly observe in data. Note that Φ_t has taken the place of the desired wage rate.

The transition laws are:

$$w_t^* = v w_{t-1}^* + \varepsilon_{w^* t} \quad (65)$$

$$\hat{w}_t = \rho_w \hat{w}_{t-1} + \varepsilon_{\hat{w} t} \quad (66)$$

The error terms are restricted:

$$\varepsilon_{\pi t}^w \sim N(0, \sigma_\pi^2) \quad (67)$$

$$\varepsilon_{w^* t} \sim N(0, \sigma_{w^*}^2) \quad (68)$$

$$\varepsilon_{\hat{w} t} \sim N(0, \sigma_{\hat{w}}^2) \quad (69)$$

4.8.1 Calibration of the Kalman-filter

It is necessary to perform calibration of the parameters and the remaining coefficients in the observation equations. It is also necessary to set initial conditions for the variances in the transition laws based on economic theory and previous research.

The parameter v measures the persistence of the desired wage rate over time. If v approaches 1, then the desired wage rate becomes a simple random walk. I undertake an intermediary step to estimate the magnitude of this persistence parameter by using a filtering method. The following table presents the Kalman-filtered iteration using state space modeling (the `sspace` method in Stata SE13.1).

Table 9: Kalman-filter on the AR(1) process of the desired wage

	Coef	Std. Err.	z	P> z	95 % CI
w_{t-1}^*	0.8421359	0.0813962	10.35	0.000	[0.6826022, 1.001669]
N	44				
Wald Chi2(1)	103.84				
Prob > Chi2	0.000				
Log-likelihood	111.24167				

We can see from Table 9 that the test gives a coefficient significant at any conventional level. It thus seems reasonable to assume that the persistence parameter $v = 0.842$. Values of the variances are set to $\sigma_\pi^2 = 0.9$ $\sigma_{w^*}^2 = 0.4$ $\sigma_w^2 = 0.1$. They compare the volatility of the desired wage rate with respect to trend, and it is reasonable to assume that the volatility of the desired wage rate be much higher than the volatility of the trend. Here, it is assumed to be four times larger.

4.9 Results of the Kalman-filter approach

Also with Kalman-filter we get forward dominance. The coefficients sum to 0.949 which is close to the theoretical 1. The iteration process needed 24 iterations to find a converging solution, and the program reported that the solution with the initialization described earlier was unique. This is a strong evidence of the Kalman-filter actually finding the unobserved component and reporting significant and interpretable results. The coefficients comparable to the GIVE and GMM approaches (lead and lagged inflation) are not significantly different in the Kalman-filter approach.

The forcing variable in the GMM approach, namely the unobserved component "desired wage rate" is significant at any conventional level of significance (p-value 0.0000). This is a strong result, and the coefficient is positive which is in line with the theory.

Table 10: Kalman-filter on the Wage-NKPC using annual data. Denominator man-year, full sample

	KF (1990-2014)
$\mathbb{E}\pi_{t+1}$	0.569591** (0.258732)
π_{t-1}	0.379184* (0.227092)
$(w_t^* - w_t)$	0.999998*** (2.49E-06)
N	40
Log likelihood	-95.65469
Akaike information criterion	-4.373399
Schwarz criterion	-4.122633
Hannan-Quinn criterion	-4.282084

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The log likelihood is quite high, indicating that the model is fitting quite well. Note that negative numbers closer to zero indicates a better fitting model. The Akaike information criterion is:

$$AIC = \ln \frac{SSR}{N} + \frac{2K}{N} \quad (70)$$

where N is the number of observations, SSR is the sum of squared residuals and K is the number of regression coefficients. Since the information criterions all are negative, it must mean that the sum of squared residuals is so small we are taking the logarithm of a number between 0 and 1. Hence, the model fits well.

4.9.1 Actual versus fundamental inflation with the Kalman-filter

Figure 6 shows the relationship between the fundamental inflation and the actual inflation using the Kalman-filter method. Comparing this to Figure 4, we can see that the Kalman-filter "smooths out" the pattern better than the GMM method. The Kalman-filter generally performs better than GMM in tracking the actual inflation. It is at least clear from the figure and estimation output that the Kalman-filter succeeds in predicting how the unobservable component affects the wage inflation, both since the coefficient is significant at any level and since the figure shows a reasonable relationship to the actual inflation. The correlation between

the fundamental inflation and the actual inflation was estimated to be 0.9064 which is a very good fit. The AR(1)-coefficient of the desired wage rate was estimated to be $\rho = 0.907$ in (26). We then get $\tilde{\alpha}_{b,1} = 0.554$ and $\tilde{\alpha}_{b,2} = 1.202$ in (21) using the estimated coefficients of the Kalman-filter method in Table 10.

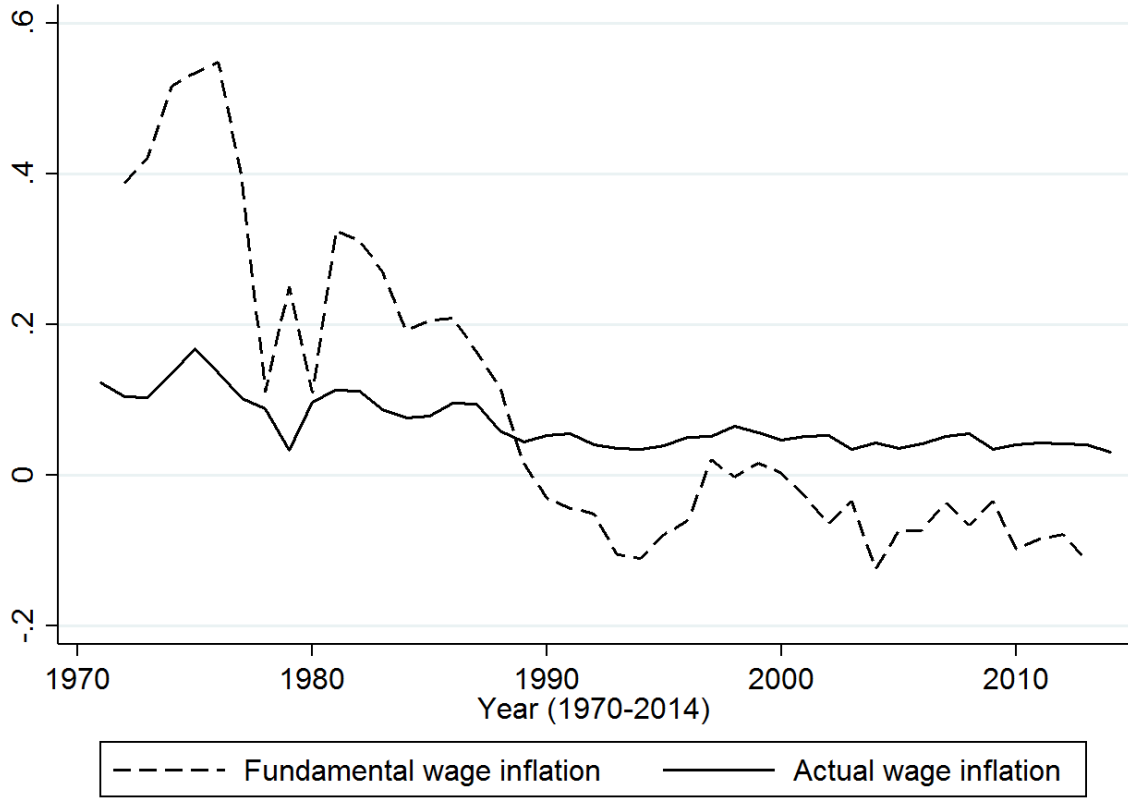


Figure 6: Inflation: Actual vs. Fundamental, Kalman-filter predicting fundamental inflation.

4.10 Comparison of sectors

Appendix B reports results for the industry sector, the total economy and the public sector. This section will discuss how those sectors compare to the Norwegian mainland sector. This is an interesting discussion because it can give an indication of the robustness of the results and possible differences in the driving of forces of inflation.

4.10.1 The total economy

As we see in Table B.1 and Table B.2, the theoretical result of forward dominance is preserved also for the total economy. Both coefficients on the lead and lagged inflation are almost identical to the ones for the mainland economy in the GIVE approach, and fairly close in the GMM approach. The coefficient on consumption has switched sign. This is a possible sign of weakness in the specification. However, the coefficient is insignificant both for the mainland economy and for the total economy. The rest of the coefficients seem to reflect the results we had for the mainland economy. The statistical significance of the marginal cost term has vaporized in the GMM approach, which is unfortunate. This likely stems from the fact that the petroleum industry is now added and this sector follows a quite different pattern in the wage setting than the rest of the economy. Note that, in line with the results for mainland Norway, the signs on the forcing variables are jointly significant. Thus, a reasonable explanation for the differences found could be the high variance in the wages in the petroleum industry.

4.10.2 The manufacturing sector

The manufacturing sector seem to have a higher degree of forward dominance than all the other sectors. Evident from Table B.3 and Table B.4, we see that this sector also has a sum of coefficients on lead and lagged inflation exceeding 1 by a significant amount. We can reject the null of homogeneity at any level of significance in both GIVE and GMM (p-value 0.3544 and 0.5248 respectively). Again, however, all the forcing variables are jointly significant at any conventional level of significance. Forward dominance in the manufacturing sector is perhaps not a surprising result, as this sector "leads" the wage negotiations in Norway.

4.10.3 The public sector

The public sector has a pattern quite similar to the mainland economy. Table B.5 and Table B.6 show forward dominance and coefficients on lead and lagged inflation summing closely to 1. We once again get coefficients on the forcing variables which are jointly significant at any conventional level of significance both in GIVE and GMM. The Sargan-Hansen test indicates a misspecified model for the public sector. This is a puzzle, but may be due to manufacturing sector productivity being too closely linked to the public sector wages and thus is not a valid instrumental variable.

5 Concluding remarks

In this master's thesis, I have studied the Hybrid Wage New Keynesian Phillips Curve and how to model the dynamics of wage inflation. The thesis has made use of three econometric estimation methods; GIVE, GMM and the Kalman-filter. I have shown that all three methods perform reasonably well on annual data. The Kalman-filter performs exceptionally well in the sense that it allows for a recursive estimation of an unobserved component rather than remodeling the system with new exogenous variables. It is also evident that the Kalman-filter does the better job in tracking actual inflation, with a markedly higher correlation than obtained by GMM.

The results suggest that the Hybrid Wage New Keynesian Phillips Curve displays reasonable coefficient estimates of both the expectations variable and the lagged inflation term, regardless of estimation method. GIVE and GMM relies on a re-specification of the model using the "pressure indicator" and the "ability to pay wages", whereas the Kalman-filter provides an estimation method which allows to directly use a model with unobservable components and achieve reasonable results. All estimation methods return reasonable coefficients on the lead and lagged inflation, where the coefficients sum closely to 1 for all methods. The results are robust to various alterations, although the sample size is too small to provide a reasonable subsample test. The empirical work shows that forward-looking behavior is dominant across all estimation methods and in all samples, which is in line with earlier research such as Bårdsen et al. (2004), Galí and Gertler (1999) and Nymoen et al. (2012). An important discovery is that even if the implied

size of the backward-looking wage setters group is small, it is statistically significant across specifications. The forcing variables are quantitatively important and jointly significant in almost all specifications and samples.

The fundamental wage inflation versus actual wage inflation plots are evidently reporting a somewhat weak fit for all estimation procedures, perhaps except the Kalman-filter which at least shows signs of high correlation. The fundamental solution is, however, based on an assumption of strongly exogenous forcing variables, which may not be entirely realistic or consistent with the idea that inflation is a variable that feed-back to several important macroeconomic variables. On the other hand, it is the same assumption that Galí and Gertler (1999) use in their seminal papers that emphasize the high degree of fit of the NKPC in the US. If the wage-NKPC is cast into a larger DSGE framework, the results could possibly improve.

On the balance of evidence, I am inclined by the econometric analysis to believe it is worth pursuing staggered wages within the context of DSGE models. This econometric evaluation has at least not disproven staggered wages as a driving force of inflation in such models. The model framework and the econometric analysis can readily be applied to quarterly data, and future work should apply such improved data sets.

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Appendix A - Output for the Mainland Economy

Table A.1: GIVE, full sample

IV (2SLS) estimation

Estimates efficient for homoskedasticity only

Statistics robust to heteroskedasticity

		Number of obs =	40
		F(7, 31) =	195.52
		Prob > F =	0.0000
Total (centered) SS	=	Centered R2	= 0.8691
Total (uncentered) SS	=	Uncentered R2	= 0.9679
Residual SS	=	Root MSE	= .01289

	pi	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
pi_lead		.6185503	.1231566	5.02	0.000	.3771677 .8599329
pi_1		.5015902	.1046326	4.79	0.000	.2965141 .7066663
c		-.0346564	.0327188	-1.06	0.289	-.098784 .0294712
dc		-.2344918	.0938022	-2.50	0.012	-.4183408 -.0506429
l		-.0215806	.0430473	-0.50	0.616	-.1059517 .0627906
A		.0636044	.0633642	1.00	0.315	-.0605871 .1877959
mc		.0837332	.0782529	1.07	0.285	-.0696397 .2371062

Underidentification test (Kleibergen-Paap rk LM statistic): 15.400
Chi-sq(5) P-val = 0.0088

Weak identification test (Cragg-Donald Wald F statistic): 11.019
(Kleibergen-Paap rk Wald F statistic): 11.210

Stock-Yogo weak ID test critical values:

5% maximal IV relative bias	18.37
10% maximal IV relative bias	10.83
20% maximal IV relative bias	6.77
30% maximal IV relative bias	5.25
10% maximal IV size	26.87
15% maximal IV size	15.09
20% maximal IV size	10.98
25% maximal IV size	8.84

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 5.915
Chi-sq(4) P-val = 0.2056

Instrumented: pi_lead
Included instruments: pi_1 c dc l A mc
Excluded instruments: import_inf logarb dffA pi_2 pi_3
Partialled-out: D88 D89
nb: small-sample adjustments account for partialled-out variables

Table A.2: GMM, full sample

2-Step GMM estimation

Estimates efficient for arbitrary heteroskedasticity
Statistics robust to heteroskedasticity

		Number of obs =	40
		F(7, 31) =	289.70
		Prob > F =	0.0000
Total (centered) SS	=	Centered R2 =	0.8615
Total (uncentered) SS	=	Uncentered R2 =	0.9661
Residual SS	=	Root MSE =	.01326

	pi	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
pi_lead		.6583463	.1120379	5.88	0.000	.438756	.8779366
pi_1		.3794636	.0754797	5.03	0.000	.2315261	.5274012
c		-.0601222	.0273201	-2.20	0.028	-.1136687	-.0065758
dc		-.1751414	.0672669	-2.60	0.009	-.306982	-.0433007
l		.011515	.0308193	0.37	0.709	-.0488896	.0719197
A		.053711	.0493508	1.09	0.276	-.0430147	.1504368
mc		.1590083	.0685251	2.32	0.020	.0247016	.293315

Underidentification test (Kleibergen-Paap rk LM statistic): 15.400
Chi-sq(5) P-val = 0.0088

Weak identification test (Cragg-Donald Wald F statistic): 11.019
(Kleibergen-Paap rk Wald F statistic): 11.210
Stock-Yogo weak ID test critical values: 5% maximal IV relative bias 18.37
10% maximal IV relative bias 10.83
20% maximal IV relative bias 6.77
30% maximal IV relative bias 5.25
10% maximal IV size 26.87
15% maximal IV size 15.09
20% maximal IV size 10.98
25% maximal IV size 8.84

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 5.915
Chi-sq(4) P-val = 0.2056

Instrumented: pi_lead
Included instruments: pi_1 c dc l A mc
Excluded instruments: import_inf logarb dffA pi_2 pi_3
Partialled-out: D88 D89
nb: small-sample adjustments account for
partialled-out variables

Table A.3: GIVE, full sample, including two additional lags

IV (2SLS) estimation

Estimates efficient for homoskedasticity only

Statistics robust to heteroskedasticity

		Number of obs =	40
		F(9, 29) =	194.25
		Prob > F =	0.0000
Total (centered) SS	=	Centered R2	= 0.8738
Total (uncentered) SS	=	Uncentered R2	= 0.9691
Residual SS	=	Root MSE	= .01266

		Robust				
	pi	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
	+					
pi_lead		.5830185	.1336142	4.36	0.000	.3211395 .8448976
pi_1		.5117298	.1451025	3.53	0.000	.227334 .7961256
c		-.0318606	.033886	-0.94	0.347	-.0982759 .0345546
dc		-.2047101	.1001063	-2.04	0.041	-.400915 -.0085053
l		.0080816	.0336183	0.24	0.810	-.057809 .0739722
A		.023479	.0603546	0.39	0.697	-.0948139 .1417719
mc		.0391777	.1088472	0.36	0.719	-.1741589 .2525143
pi_2		.0033119	.1166105	0.03	0.977	-.2252405 .2318643
pi_3		-.1112677	.1563655	-0.71	0.477	-.4177385 .195203

Underidentification test (Kleibergen-Paap rk LM statistic): 11.884
Chi-sq(3) P-val = 0.0078

Weak identification test (Cragg-Donald Wald F statistic): 15.769
(Kleibergen-Paap rk Wald F statistic): 14.056

Stock-Yogo weak ID test critical values:

5% maximal IV relative bias	13.91
10% maximal IV relative bias	9.08
20% maximal IV relative bias	6.46
30% maximal IV relative bias	5.39
10% maximal IV size	22.30
15% maximal IV size	12.83
20% maximal IV size	9.54
25% maximal IV size	7.80

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 5.538
Chi-sq(2) P-val = 0.0627

Instrumented: pi_lead
Included instruments: pi_1 c dc l A mc pi_2 pi_3
Excluded instruments: import_inf logarb dffa
Partialled-out: D88 D89
nb: small-sample adjustments account for partialled-out variables

Table A.4: GMM, full sample, including two additional lags

2-Step GMM estimation

Estimates efficient for arbitrary heteroskedasticity

Statistics robust to heteroskedasticity

		Number of obs =	40
		F(9, 29) =	238.39
		Prob > F =	0.0000
Total (centered) SS	=	Centered R2	= 0.8614
Total (uncentered) SS	=	Uncentered R2	= 0.9660
Residual SS	=	Root MSE	= .01327

		Robust				
	pi	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
	+					
	pi_lead	.674584	.1263194	5.34	0.000	.4270024 .9221655
	pi_1	.3249166	.1186494	2.74	0.006	.092368 .5574651
	c	-.054682	.02999	-1.82	0.068	-.1134613 .0040973
	dc	-.1811009	.095771	-1.89	0.059	-.3688086 .0066069
	l	.0198393	.0332092	0.60	0.550	-.0452496 .0849282
	A	.0380331	.0595345	0.64	0.523	-.0786523 .1547186
	mc	.1438625	.0939184	1.53	0.126	-.0402143 .3279393
	pi_2	.0634476	.1126693	0.56	0.573	-.1573802 .2842754
	pi_3	-.0670669	.1240668	-0.54	0.589	-.3102333 .1760995

Underidentification test (Kleibergen-Paap rk LM statistic): 11.884
Chi-sq(3) P-val = 0.0078

Weak identification test (Cragg-Donald Wald F statistic): 15.769
(Kleibergen-Paap rk Wald F statistic): 14.056

Stock-Yogo weak ID test critical values:

5% maximal IV relative bias	13.91
10% maximal IV relative bias	9.08
20% maximal IV relative bias	6.46
30% maximal IV relative bias	5.39
10% maximal IV size	22.30
15% maximal IV size	12.83
20% maximal IV size	9.54
25% maximal IV size	7.80

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 5.538
Chi-sq(2) P-val = 0.0627

Instrumented: pi_lead
Included instruments: pi_1 c dc l A mc pi_2 pi_3
Excluded instruments: import_inf logarb dffa
Partialled-out: D88 D89
nb: small-sample adjustments account for partialled-out variables

Table A.5: GIVE, full sample, using man-hours as denominator value

IV (2SLS) estimation

Estimates efficient for homoskedasticity only

Statistics robust to heteroskedasticity

		Number of obs =	40
		F(7, 31) =	185.39
		Prob > F =	0.0000
Total (centered) SS	=	Centered R2	= 0.8897
Total (uncentered) SS	=	Uncentered R2	= 0.9731
Residual SS	=	Root MSE	= .01271

		Robust				
	pi	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
	+					
pi_lead		.7039462	.1235917	5.70	0.000	.4617109 .9461815
pi_1		.6285056	.105082	5.98	0.000	.4225486 .8344626
c		-.1158023	.0647948	-1.79	0.074	-.2427978 .0111932
dc		-.3362794	.1295164	-2.60	0.009	-.5901268 -.082432
l		-.0428329	.0308061	-1.39	0.164	-.1032118 .017546
A		.1744828	.0973317	1.79	0.073	-.0162837 .3652494
mc		.1380799	.0935778	1.48	0.140	-.0453292 .321489

Underidentification test (Kleibergen-Paap rk LM statistic): 20.793
Chi-sq(5) P-val = 0.0009

Weak identification test (Cragg-Donald Wald F statistic): 5.920
(Kleibergen-Paap rk Wald F statistic): 4.573

Stock-Yogo weak ID test critical values:

5% maximal IV relative bias	18.37
10% maximal IV relative bias	10.83
20% maximal IV relative bias	6.77
30% maximal IV relative bias	5.25
10% maximal IV size	26.87
15% maximal IV size	15.09
20% maximal IV size	10.98
25% maximal IV size	8.84

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 1.414
Chi-sq(4) P-val = 0.8417

Instrumented: pi_lead
Included instruments: pi_1 c dc l A mc
Excluded instruments: import_inf logarb dffA pi_2 pi_3
Partialled-out: D88 D89
nb: small-sample adjustments account for partialled-out variables

Table A.6: GMM, full sample, using man-hours as denominator value

2-Step GMM estimation

Estimates efficient for arbitrary heteroskedasticity

Statistics robust to heteroskedasticity

		Number of obs =	40
		F(7, 31) =	234.56
		Prob > F =	0.0000
Total (centered) SS	=	Centered R2	= 0.8879
Total (uncentered) SS	=	Uncentered R2	= 0.9727
Residual SS	=	Root MSE	= .01282

		Robust				
	pi	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
	+					
	pi_lead	.7046097	.1122273	6.28	0.000	.4846482 .9245712
	pi_1	.5859449	.0865108	6.77	0.000	.4163868 .755503
	c	-.0842345	.0568701	-1.48	0.139	-.1956979 .0272289
	dc	-.2614737	.1061166	-2.46	0.014	-.4694584 -.053489
	l	-.0281113	.0242612	-1.16	0.247	-.0756623 .0194398
	A	.1259126	.0825178	1.53	0.127	-.0358194 .2876445
	mc	.1444957	.0793308	1.82	0.069	-.0109898 .2999812

Underidentification test (Kleibergen-Paap rk LM statistic): 20.793
Chi-sq(5) P-val = 0.0009

Weak identification test (Cragg-Donald Wald F statistic): 5.920
(Kleibergen-Paap rk Wald F statistic): 4.573

Stock-Yogo weak ID test critical values:

5% maximal IV relative bias	18.37
10% maximal IV relative bias	10.83
20% maximal IV relative bias	6.77
30% maximal IV relative bias	5.25
10% maximal IV size	26.87
15% maximal IV size	15.09
20% maximal IV size	10.98
25% maximal IV size	8.84

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 1.414
Chi-sq(4) P-val = 0.8417

Instrumented: pi_lead
Included instruments: pi_1 c dc l A mc
Excluded instruments: import_inf logarb dffA pi_2 pi_3
Partialled-out: D88 D89
nb: small-sample adjustments account for
partialled-out variables

Appendix B - Output for other sectors

Table B.1: GIVE, full sample, Total Norwegian economy

IV (2SLS) estimation

Estimates efficient for homoskedasticity only

Statistics robust to heteroskedasticity

		Number of obs =	40
		F(7, 31) =	314.98
		Prob > F =	0.0000
Total (centered) SS	=	Centered R2	= 0.8745
Total (uncentered) SS	=	Uncentered R2	= 0.9698
Residual SS	=	Root MSE	= .01238

	pi	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
	pi_lead	.6038785	.1062467	5.68	0.000	.3956389 .8121182
	pi_1	.5098203	.1108756	4.60	0.000	.2925081 .7271326
	c	.0106399	.022212	0.48	0.632	-.0328949 .0541747
	dc	-.2364611	.0898084	-2.63	0.008	-.4124823 -.0604399
	l	-.0037558	.0166127	-0.23	0.821	-.036316 .0288045
	A	.0136172	.023949	0.57	0.570	-.033322 .0605564
	mc	.0305865	.0299286	1.02	0.307	-.0280725 .0892455

Underidentification test (Kleibergen-Paap rk LM statistic): 14.266
Chi-sq(5) P-val = 0.0140

Weak identification test (Cragg-Donald Wald F statistic): 12.198

(Kleibergen-Paap rk Wald F statistic): 13.806

Stock-Yogo weak ID test critical values:

5% maximal IV relative bias	18.37
10% maximal IV relative bias	10.83
20% maximal IV relative bias	6.77
30% maximal IV relative bias	5.25
10% maximal IV size	26.87
15% maximal IV size	15.09
20% maximal IV size	10.98
25% maximal IV size	8.84

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 7.728
Chi-sq(4) P-val = 0.1021

Instrumented: pi_lead
Included instruments: pi_1 c dc l A mc
Excluded instruments: import_inf logarb dffA pi_2 pi_3
Partialled-out: D88 D89
nb: small-sample adjustments account for
partialled-out variables

Table B.2: GMM, full sample, Total Norwegian economy

2-Step GMM estimation

Estimates efficient for arbitrary heteroskedasticity

Statistics robust to heteroskedasticity

		Number of obs =	40
		F(7, 31) =	342.71
		Prob > F =	0.0000
Total (centered) SS	=	Centered R2	= 0.8707
Total (uncentered) SS	=	Uncentered R2	= 0.9688
Residual SS	=	Root MSE	= .01257

		Robust				
pi	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
pi_lead	.6669038	.0974233	6.85	0.000	.4759576	.85785
pi_1	.4575476	.0765632	5.98	0.000	.3074865	.6076087
c	.0113511	.0204735	0.55	0.579	-.0287763	.0514785
dc	-.209209	.0681434	-3.07	0.002	-.3427675	-.0756504
l	-.0042698	.01554	-0.27	0.783	-.0347276	.0261879
A	.0094131	.0171164	0.55	0.582	-.0241344	.0429607
mc	.038638	.0244014	1.58	0.113	-.0091878	.0864638

Underidentification test (Kleibergen-Paap rk LM statistic): 14.266
Chi-sq(5) P-val = 0.0140

Weak identification test (Cragg-Donald Wald F statistic): 12.198
(Kleibergen-Paap rk Wald F statistic): 13.806

Stock-Yogo weak ID test critical values:

5% maximal IV relative bias	18.37
10% maximal IV relative bias	10.83
20% maximal IV relative bias	6.77
30% maximal IV relative bias	5.25
10% maximal IV size	26.87
15% maximal IV size	15.09
20% maximal IV size	10.98
25% maximal IV size	8.84

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 7.728
Chi-sq(4) P-val = 0.1021

Instrumented: pi_lead
Included instruments: pi_1 c dc l A mc
Excluded instruments: import_inf logarb dffA pi_2 pi_3
Partialled-out: D88 D89
nb: small-sample adjustments account for partialled-out variables

Table B.3: GIVE, full sample, Manufacturing sector

IV (2SLS) estimation

Estimates efficient for homoskedasticity only

Statistics robust to heteroskedasticity

		Number of obs =	40
		F(7, 31) =	112.16
		Prob > F =	0.0000
Total (centered) SS	=	Centered R2	= 0.8061
Total (uncentered) SS	=	Uncentered R2	= 0.9518
Residual SS	=	Root MSE	= .01584

	pi	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
	pi_lead	.843319	.1689784	4.99	0.000	.5121275 1.17451
	pi_1	.370331	.133371	2.78	0.005	.1089286 .6317334
	c	.0247558	.0286893	0.86	0.388	-.0314742 .0809858
	dc	-.0486106	.0951863	-0.51	0.610	-.2351723 .1379511
	l	-.0254919	.033018	-0.77	0.440	-.0902059 .0392221
	A	-.0186415	.0579608	-0.32	0.748	-.1322426 .0949597
	mc	.121022	.0701247	1.73	0.084	-.01642 .258464

Underidentification test (Kleibergen-Paap rk LM statistic): 8.571
Chi-sq(5) P-val = 0.1274

Weak identification test (Cragg-Donald Wald F statistic): 6.419
(Kleibergen-Paap rk Wald F statistic): 6.800

Stock-Yogo weak ID test critical values:

5% maximal IV relative bias	18.37
10% maximal IV relative bias	10.83
20% maximal IV relative bias	6.77
30% maximal IV relative bias	5.25
10% maximal IV size	26.87
15% maximal IV size	15.09
20% maximal IV size	10.98
25% maximal IV size	8.84

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 2.177
Chi-sq(4) P-val = 0.7032

Instrumented: pi_lead
Included instruments: pi_1 c dc l A mc
Excluded instruments: import_inf logarb pi_2 pi_3
Partialled-out: D88 D89
nb: small-sample adjustments account for
partialled-out variables

Table B.4: GMM, full sample, Manufacturing sector

2-Step GMM estimation

Estimates efficient for arbitrary heteroskedasticity

Statistics robust to heteroskedasticity

		Number of obs =	40
		F(7, 31) =	183.76
		Prob > F =	0.0000
Total (centered) SS	=	Centered R2	= 0.8105
Total (uncentered) SS	=	Uncentered R2	= 0.9529
Residual SS	=	Root MSE	= .01565

	pi	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
	pi_lead	.7888179	.1362102	5.79	0.000	.5218507 1.055785
	pi_1	.3221217	.0982303	3.28	0.001	.1295938 .5146496
	c	.0217929	.0259915	0.84	0.402	-.0291494 .0727353
	dc	-.0538917	.0904341	-0.60	0.551	-.2311394 .1233559
	l	-.0207978	.0295746	-0.70	0.482	-.078763 .0371674
	A	-.019851	.0505666	-0.39	0.695	-.1189598 .0792578
	mc	.1135555	.0615423	1.85	0.065	-.0070651 .2341762

Underidentification test (Kleibergen-Paap rk LM statistic): 8.571
Chi-sq(5) P-val = 0.1274

Weak identification test (Cragg-Donald Wald F statistic): 6.419
(Kleibergen-Paap rk Wald F statistic): 6.800

Stock-Yogo weak ID test critical values:

5% maximal IV relative bias	18.37
10% maximal IV relative bias	10.83
20% maximal IV relative bias	6.77
30% maximal IV relative bias	5.25
10% maximal IV size	26.87
15% maximal IV size	15.09
20% maximal IV size	10.98
25% maximal IV size	8.84

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 2.177
Chi-sq(4) P-val = 0.7032

Instrumented: pi_lead
Included instruments: pi_1 c dc l A mc
Excluded instruments: import_inf logarb pi_2 pi_3
Partialled-out: D88 D89
nb: small-sample adjustments account for partialled-out variables

Table B.5: GIVE, full sample, Public sector

IV (2SLS) estimation

Estimates efficient for homoskedasticity only

Statistics robust to heteroskedasticity

		Number of obs =	40
		F(7, 31) =	196.35
		Prob > F =	0.0000
Total (centered) SS	=	Centered R2 =	0.8617
Total (uncentered) SS	=	Uncentered R2 =	0.9669
Residual SS	=	Root MSE =	.01204

	pi	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
	pi_lead	.5976268	.1069368	5.59	0.000	.3880344 .8072192
	pi_1	.5029074	.1078999	4.66	0.000	.2914274 .7143873
	c	-.0428926	.0414693	-1.03	0.301	-.124171 .0383858
	dc	-.1477132	.0728942	-2.03	0.043	-.2905831 -.0048432
	l	.0603721	.056457	1.07	0.285	-.0502817 .1710259
	A	.0337135	.0384871	0.88	0.381	-.0417198 .1091468
	mc	.3735231	.3452136	1.08	0.279	-.3030832 1.050129

Underidentification test (Kleibergen-Paap rk LM statistic): 8.774
Chi-sq(5) P-val = 0.1184

Weak identification test (Cragg-Donald Wald F statistic): 4.288

(Kleibergen-Paap rk Wald F statistic): 3.938

Stock-Yogo weak ID test critical values:

5% maximal IV relative bias	18.37
10% maximal IV relative bias	10.83
20% maximal IV relative bias	6.77
30% maximal IV relative bias	5.25
10% maximal IV size	26.87
15% maximal IV size	15.09
20% maximal IV size	10.98
25% maximal IV size	8.84

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 12.568
Chi-sq(4) P-val = 0.0136

Instrumented: pi_lead
Included instruments: pi_1 c dc l A mc
Excluded instruments: import_inf logarb dffA pi_2 pi_3
Partialled-out: D88 D89
nb: small-sample adjustments account for partialled-out variables

Table B.6: GMM, full sample, Public sector

2-Step GMM estimation

Estimates efficient for arbitrary heteroskedasticity

Statistics robust to heteroskedasticity

		Number of obs =	40
		F(7, 31) =	260.14
		Prob > F =	0.0000
Total (centered) SS	=	Centered R2 =	0.8572
Total (uncentered) SS	=	Uncentered R2 =	0.9658
Residual SS	=	Root MSE =	.01223

		Robust				
pi	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
pi_lead	.609287	.1010654	6.03	0.000	.4112025	.8073715
pi_1	.5657783	.0815706	6.94	0.000	.4059027	.7256538
c	-.016346	.0281623	-0.58	0.562	-.0715431	.0388511
dc	-.1467844	.0640918	-2.29	0.022	-.272402	-.0211668
l	.017796	.0396565	0.45	0.654	-.0599294	.0955213
A	-.0058544	.0329629	-0.18	0.859	-.0704605	.0587518
mc	.0197037	.2758407	0.07	0.943	-.5209342	.5603415

Underidentification test (Kleibergen-Paap rk LM statistic): 8.774
Chi-sq(5) P-val = 0.1184

Weak identification test (Cragg-Donald Wald F statistic): 4.288
(Kleibergen-Paap rk Wald F statistic): 3.938

Stock-Yogo weak ID test critical values:

5% maximal IV relative bias	18.37
10% maximal IV relative bias	10.83
20% maximal IV relative bias	6.77
30% maximal IV relative bias	5.25
10% maximal IV size	26.87
15% maximal IV size	15.09
20% maximal IV size	10.98
25% maximal IV size	8.84

Source: Stock-Yogo (2005). Reproduced by permission.

NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.

Hansen J statistic (overidentification test of all instruments): 12.568
Chi-sq(4) P-val = 0.0136

Instrumented: pi_lead
Included instruments: pi_1 c dc l A mc
Excluded instruments: import_inf logarb dffA pi_2 pi_3
Partialled-out: D88 D89
nb: small-sample adjustments account for partialled-out variables